ISTANBUL TECHNICAL UNIVERSITY ★ FACULTY OF AERONAUTICS AND ASTRONAUTICS

ROLLING AIRFRAME ROCKET MODELLING

GRADUATION PROJECT

Akın ÇATAK

Department of Aeronautical Engineering

Thesis Advisor: Asst. Prof. Emre KOYUNCU

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Akın ÇATAK, student of ITU Faculty of Aeronautics and Astronauticsstudent ID **110150039**, successfully defended the **graduation** entitled "**ROLLING AIRFRAME ROCKET MODELLING**", which he/she prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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Date of Submission : 13 July 2020

Date of Defense : 21 July 2020

To my family,

FOREWORD

I would like to thank, my thesis advisor Mr. Asst. Professor Emre Koyuncu, my friends Ahmet Talha ÇETİN, Oğuzhan KÖSE, and Esra Demir due to their help during the thesis.

July 2020 Akın ÇATAK

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ABBREVIATIONS

DOF	**:** Degree of Freedom
ECEF	**:** Earth Centered Earth Fixed
ECI	**:** Earth Centered Inertial
EGM	**:** Earth Gravitational Model
FCG	**:** Center of Gravity of Fuel
$\mathbf{FI_{xx}}$	**:** Fuel's Moment of Inertia in X Axis
$\mathbf{FI_{yy}}$	**:** Fuel's Moment of Inertia in Y Axis
$\mathbf{FI_{zz}}$	**:** Fuel's Moment of Inertia in Z Axis
FM	**:** Fuel Mass
OCG	**:** Center of Gravity of Oxidizer
$\mathbf{OI_{xx}}$	**:** Oxidizer's Moment of Inertia in X Axis
$\mathbf{OI_{yy}}$	**:** Oxidizer's Moment of Inertia in Y Axis
$\mathbf{OI_{zz}}$	: Oxidizer's Moment of Inertia in Z Axis
OM	**:** Oxidizer Mass
RCG	**:** Center of Gravity of Empty Rocket
$\mathbf{RI_{xx}}$	**:** Empty Rocket's Moment of Inertia in X Axis
$\mathbf{RI_{yy}}$	**:** Empty Rocket's Moment of Inertia in Y Axis
$\mathbf{RI_{zz}}$	**:** Empty Rocket's Moment of Inertia in Z Axis
RM	**:** Empty Rocket's Mass
TCG	**:** Total Center of Gravity
$\mathbf{TI_{xx}}$	**:** Total Moment of Inertia in X Axis
$\mathbf{TI_{yy}}$	**:** Total Moment of Inertia in Y Axis
$\mathbf{TI_{zz}}$	**:** Total Moment of Inertia in Z Axis
WGS	**:** World Geodetic System

SYMBOLS

$\hat{\boldsymbol{\imath}}_e, \hat{\boldsymbol{\jmath}}_e, \hat{\boldsymbol{k}}_e$	: Basis Vectors of Earth Frame
$\hat{\boldsymbol{\imath}}_b, \hat{\boldsymbol{\jmath}}_b, \hat{\boldsymbol{k}}_b$	: Basis Vectors of Body Frame
$\boldsymbol{\omega}_E$	: Earth's Rotational Speed
$\boldsymbol{\Omega}$	: Angle between ECI and ECEF
$\boldsymbol{T}_X^Y$	: X to Y Transformation Matrix
$\boldsymbol{\alpha}$	: Angle of Attack
$\boldsymbol{\beta}$	: Beta Angle
$\boldsymbol{\alpha}'$	: Total Angle of Attack
$\boldsymbol{\phi}'$	: Aerodynamic Roll Angle
$\boldsymbol{p}, \boldsymbol{q}, \boldsymbol{r}$	: Angular Rates
$\boldsymbol{\phi}, \boldsymbol{\theta}, \boldsymbol{\psi}$	: Attitude Angles
$\boldsymbol{u}, \boldsymbol{v}, \boldsymbol{w}$	: Velocity Components
$\boldsymbol{q}_0, \boldsymbol{q}_1, \boldsymbol{q}_2, \boldsymbol{q}_3$	: Quaternion Components
$\boldsymbol{l}$	: Angular Momentum
$\boldsymbol{I}$	: Moment of Inertia
$\boldsymbol{Q}$	: Dynamic Pressure
$\boldsymbol{S}$	: Reference Area
$\mathbf{d}$	: Reference Length
$\mathbf{F}_A, \mathbf{F}_P, \mathbf{F}_g$	: Aerodynamic, Propulsive, and Gravitational Force
$\boldsymbol{M}_A$	: Aerodynamic Moment
$\boldsymbol{T}$	: Temperature
$\boldsymbol{P}$	: Pressure
$\boldsymbol{\rho}$	: Density
$\boldsymbol{c}$	: Speed of Sound
$\boldsymbol{g}$	: Gravity Vector

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ROLLING AIRFRAME ROCKET MODELLING

SUMMARY

In order to model a rocket moving with constant spin rate, the rocket's equations of motion must be obtained. For this, firstly coordinate frames were examined. After analyzing the inertial frames and reference frames, the Flat Earth Inertia axis set was chosen as the Inertial frame set, where the world is regarded as flat. The Body frame and the Non-Rolling Body frame are selected as reference frames in which equations of motions are written. Then, two separate models created. After the reference frame acceptance, the equations are written on the selected reference frames.

Thanks to the second law of Newton, known as the total force acting on an object equal to the change in the momentum of that object, rockets equations of motion were obtained. The forces acting on the object in motion equations are three: aerodynamic, gravity and thrust. Aerodynamic forces are found using the Missile DATCOM software. The gravitational force is determined by using the instantaneous weight of the rocket and the current gravitational acceleration, while the thrust force is obtained with the help of the data in the literature.

The modeled rocket has 6 degrees of freedom. Three of them come from linear velocity in three directions in translation equations, while the remaining 3 degrees of freedom come from angular velocity expressions in rotational equations. The 2nd Law of Euler, which helps us model the rotational motion, states that the total moments affecting an object are equal to the change in the angular momentum of that object. Moment constants provided by the Missile DATCOM application were used to calculate the moments required to create the rotational equations. Since the gravitational force passing through the center of gravity of the rocket and the thrust force generated in the axial direction did not create momentum, they did not affect the vehicle's rotational equations.

The rocket model consists of subsystems. Atmosphere properties, gravity properties, propulsion and aerodynamic properties are examples of these subsystems. Before transitioning to the computer program where the model is completed and simulated, the subsystems must be modeled. Atmosphere model covers variables such as temperature, density, pressure, speed of sound velocity and dynamic pressure. These variables take different values according to altitude. Using the US Standard Atmosphere 1976, each variable is written as a function depending on the height.

Gravity subsystem was calculated and modeled according to latitude and position vector using the gravitational potential of the world in WGS 84. The centripetal acceleration has been neglected since the Earth frame was chosen as the Inertial frame.

Aerobee 150A rocket, selected for model study, has two stages. The first stage use solid fuel and the second stage use liquid fuel. Since the first stage was neglected in the reference study in which the physical parameters of the rocket were taken, the Aerobee 150A rocket was assumed to be a single stage in the thesis study. The thrust profile of the second stage was taken constant until engine runs out.

Missile DATCOM outputs were used while creating the aerodynamic model. The outputs from Missile DATCOM were edited with a code and saved as a table in Matlab workspace. Each of these tables was used to create look up tables while creating the Simulink model.

After the rotational equations, translational equations and subsystems are modeled, the mathematical model created is transferred to the Simulink ecosystem. Simulink model, in which the relevant blocks are created for each subsystem, has been tested in 3 different situations. The first of these situations is that the body frame is taken as the rockets reference frame and rocket is not rotating. Second situation is that the body frame is taken as the rockets reference frame and rocket is rotating at constant 1 Hz in axial axis. Third and last one is that the Non-Rolling Body frame is taken as the rockets reference frame and rocket is rotating at constant 1 Hz in axial axis.

The simulation results of the created models are similar to each other. However, there are big differences compared to the results in the literature. While the reasons for some of these differences were understood, comments were made about the rest. The reason why the maximum altitude is less than the results in the literature is that the first neglected stage of the rocket. In addition, the total attack angle range, in which aerodynamic forces and moments are defined, does not cover some areas during flight. Taking the closest values for these uncovered portions distorts the moment balance in some flight regimes like entering back to the atmosphere. As a result, erroneous results are obtained.

DÖNEN BİR ROKETİN MODELLENMESİ

ÖZET

Sabit dönme hızı ile hareket eden bir roketin modellenebilmesi için roketin hareket denklemlerinin çıkarılması gerekmektedir. Bunun için öncelikle eksen takımları incelenmiştir. Atalet eksen takımı ve referans eksen takımları incelendikten sonra atalet eksen takımı olarak dünyanın düz kabul edildiği dünya atalet eksen takımı seçilmiştir. Hareket denklemlerinin yazılacağı referans eksen takımı olarak Gövde eksen takımı ve dönmeyen gövde eksen takımları ayrı iki modelde incelenmiştir. Eksen takımları kabulünden sonra hareket denklemleri seçilen eksen takımlarında yazılmıştır.

Newtonun, bir cismin üzerine etki eden toplam kuvvetin o cismin momentumundaki değişime eşit olması olarak bilinen ikinci kanunu sayesinde roketin öteleme hareket denklemleri elde edilmiştir. Hareket denklemlerinde cisme etkiyen kuvvetler aerodinamik, yer çekimi ve itki olmak üzere üç adettir. Bu kuvvetlerden aerodinamik olanlar Missile DATCOM yazılımı kullanılarak elde edilen kuvvet sabitleri sayesinde, yer çekimi kuvveti roketin anlık ağırlığı ve o anki yer çekimi ivmesi sayesinde, itki kuvveti de literatürdeki veriler ışığında hesaplanmaktadır.

Modellenen roketin 6 serbestlik derecesi vardır. Üç adedi öteleme denklemlerindeki üç yönde çizgisel hızdan gelirken, geriye kalan 3 serbestlik derecesi dönme denklemlerindeki açısal hız ifadelerinden gelmektedir. Dönme hareketerini modellememize yarayan Eulerin 2. Kanunu bir cisme etki eden toplam momentlerin, o cismin açısal momentumundaki değişime eşit olduğunu belirtmektedir. Dönme denklemlerini oluşturmak için gerekli olan momentlerin hesabı için Missile DATCOM uygulamasının sağladığı moment sabitleri kullanılmıştır. Roketin ağırlık merkezinden geçen yer çekimi kuvveti ve eksenel doğrultuda oluşan itki kuvveti moment oluşturmadıkları için aracın dönme denklemlerini etkilememiştir.

Roket modeli alt sistemlerden oluşmaktadır. Atmosfer özellikleri, yer çekimi özellikleri, itki ve aerodinamik özellikler bu alt sistemlere verilen örneklerdir. Modelin tamamlanıp simülasyonun yapılacağı bilgisayar programına geçiş yapılmadan önce alt sistemlerin modellenmesi gerekmektedir. Atmosfer modeli; sıcaklık, yoğunluk, basınç, ses hızı ve dinamik basınç gibi değişkenleri kapsamaktadır. Bu değişkenler irtifaya göre farklı değerler almaktadırlar. US Standart Atmosfer 1976 kullanılarak her bir değişken yüksekliğe bağlı birer fonksiyon olarak yazılmıştır.

Yer çekimi alt sistemi WGS 84 te bulunan dünyanın gravitasyonel potansiyeli kullanılarak enlem ve pozisyon vektörüne göre hesaplanmış ve modellenmiştir. Dünya atalet eksen takımı olarak seçildiği için merkezcil ivme ihmal edilmiştir.

Model çalışması için seçilmiş olan Aerobee 150A roketi iki kademelidir. İlk kademesi katı yakıtlı olup ikinci ikinci kademesi sıvı yakıtlıdır. Roketin fiziksel parametrelerinin alındığı referans çalışmada ilk kademe ihmal edildiği için tez çalışmasında Aerobee 150A roketi tek kademeli olarak kabul edilmiştir. İkinci kademenin itki profili yanma süresince sabit olarak alınmıştır.

Aerodinamik model oluşturulurken Missile DATCOM çıktıları kullanılmıştır. Missile DATCOM dan alınan çıktılar yazılan bir kod yardımıyla düzenlenerek Matlab workspace ine tablo olarak kaydedilmiştir. Kaydedilen tabloların her biri Simulink modeli oluşturulurken look up table ların oluşturulmasında kullanılmıştır.

Dönme ve öteleme denklemleri ile birlikte alt sistemler de modellendikten sonra oluşturulan matematiksel model Simulink ekosistemine aktarılmıştır. Her bir alt sistem için ilgili blokların oluşturulduğu Simulink modeli; roket referans eksen takımı olarak gövde eksen takımının alındığı ve dönmeyen, referans eksen takımı olarak gövde eksen takımının alındığı ve sabit dönme hızına sahip olan ve son olarak referans eksen takımı olarak dönmeyen gövde eksen takımının alındığı ve sabit dönme hızına sahip olan 3 farklı durumda test edilmiştir.

Oluşturulmuş modellerin simülasyon sonuçları birbirleriyle benzerlik göstermektedir. Fakat literatürdeki sonuçlarla arada büyük farklar bulunmaktadır. Bu farkların bazılarının sebebi anlaşılmışken geri kalanlar hakkında yorumlar yapılmıştır. Maksimum irtifanın literatürdeki sonuçlardan az çıkmasının sebebi roketin ihmal edilen ilk kademesinin yüksek bir itkiye sahip olmasıdır. Ayrıca aerodinamik kuvvet ve momentlerin tanımlı olduğu toplam hücum açısı aralığı uçuş esnasındaki bazı bölgeleri kapsamamaktadır. Bu değerler için en yakın değerlerin alınması atmosfere giriş yapılırken moment dengesini bozmakta ve bunun sonucunda hatalı sonuçlar elde edilmektedir.

1.INTRODUCTION

The dream of reaching the sky in the history of humanity has been around for a very long time. The history of the rocket, which started with fireworks and fire arrows, dates back to the centuries. The history of the first rocket-like instrument in history was recorded in Chinese and the Mongols War in 1232 [1]. In the 17th century, the work of Sir Isaac Newton opened the way for rocket works. The laws that he set out explain the working principles of rockets.

The history of modern rockets began with Konstantin Tsiolkovsky (1857-1935). Tsiolkovsky wrote a report that rockets can be used for space exploration. At the beginning of the 20th century, American Robert H. Goddard (1882-1945) started working on rockets. Having worked on solid fuel rockets, Goddard realized that the liquid fuel rockets could reach higher altitudes and realized his first successful end in March 16, 1926 [1]. He and his rocket are shown in Figure1.1. Although Goddard's first

Figure 1.1: Robert H. Goddard and his Rocket

flight has reached an altitude of 12.5 m, today is a very important step for rocket technology.

Hermann Oberth, who was then writing a book about rocket travel, was the basis for the production of German V-2 rockets used in the Second World War. After the war, many scientists, including Wernher von Braun, went to the USA or the Soviet and gave their work in those countries.

The space race, which continues as satellite launch and humanity to step on the moon, is still an important issue in our day. The number of publications related to this issue has increased in time and the states and companies that have been interested in this subject throughout the world have increased.

Before mentioning which types of rockets are, it is necessary to state the difference of the rocket from missile. Rockets are vehicles that are not guided, aiming and following ballistic trajectory once launched. The other vehicle, called missile, is guided after launching, unlike the rocket. Rockets can have engines that work with the use of air in the atmosphere called air breathing, as well as engines capable of working outside of the atmosphere called rocket engines which can be use solid or liquid fuel. The basic principle of operation is that the impulse created by the engines overcomes the weight and drag force and the rocket gains altitude.

The rocket to be used in the scope of the thesis is the Aerobee 150A model. This rocket is a sounding rocket used intensively by the USA for high atmospheric and cosmic radiation studies in the 1950s and 1960s. The Aerobee rocket, which made its final flight in 1985, consists of two stages.The first stage, the booster, is solid fuel and the second stage is liquid. However, for simplicity Aerobee 150A is assumed to be one stage meaning having liquid engine without booster. The reason for this rocket being chosen within the scope of the thesis is the fact that it has been used for nearly 40 years and the availability of data related to the rocket in the literature.

1.1. Purpose of Thesis

In this thesis, it is planned to create a nonlinear rocket model by obtaining the translational and rotational equations of motions of a spinning sounding rocket. The force and moment constants required for the rocket's translational and rotational equations will be obtained by using Missile DATCOM software. The equations for gravitational and atmospheric properties, which are among the environmental factors, will be derived in accordance with the flight regime and these equations will be explained in the necessary sections. The equations obtained will be transferred to the Simulink platform and the model created will be simulated. Model outputs for different flight situations will be examined.

1.2. Literature Review

In this thesis, 6DOF modeling and simulation of a rolling airframe rocket has been done. In addition to the 6Dof model, 3DOF and 5DOF models are also made while modeling the rocket. Basically, the model is 5 or 6 DOF according to the degrees of freedom of rotation added to the 3 DOF model which has 3 degree of freedom in translational movement. Main resource used in this regard is Zipfel (2007). For dynamic equations used in rocket modeling, Zipfel (2007), İpek (2015) and Wie (2008) were used. The main source which was used for modelling Quaternions is Diebel (2006). As resource of transformation matrices and reference frames, Lustig (2017) has been examined in addition to Zipfel (2007). The rocket, whose mathematical model was created, was chosen as the Aerobee150A using Top (2019). The reason for choosing Aerobee150A is that Top (2019) provides all the physical parameters required during the thesis. The force and moment constants of the rocket were obtained from the Missile DATCOM software by using the Blake(1998). Environmental and gravity models were created by examining US Standard Atmosphere (1976) and Wie (2008) sources, respectively. Matlab-Simulink environment, which are mostly used in model study, were chosen due to the flexibility of use as a simulation environment in addition to my knowledge in these platform. The model written in the Non-Rolling Body frame suggested in Zipfel (2007) and the model written in Body frame, which is used in most of the modeling studies, are simulated. The outputs of the models are compared with each other and with Radulescu (2016) source.

2.MATHEMATICAL MODEL

2.1. Reference Frames

Understanding the reference frames is one of the most important issues for rocket modeling. It is obvious that the position or speed of the rocket will take different values on different frames. A particular frame, in which the forces and moments used in rocket modeling are obtained, will be different from the reference frames in which rocket modeling is performed. In addition, since the frames on which the sensors and equipment to be used on the rocket are located may not be on the same frame as the body, the values they will express on the body frame will be different. In this section, certain reference frames will be explained and how to make transformations between these frames will be explained.

2.1.1ECI frame

The origin of the Earth Centered Inertial Frame (ECI) is overlapped with the center of mass of the world. This frame does not rotate when the Earth rotates. To express the axes of ECI the J2000 system is used with basis vectors $\hat{\imath}$ $\hat{\jmath}$ $\hat{k}$ given as follows. The vector of $\hat{\imath}$ is located in the equatorial plane pointing towards the vernal equinox in J2000 epoch. The $\hat{k}$ vector is normal to the equatorial plane, coincides with the Earth's axis of rotation and points to the North pole and the vector $\hat{\jmath}$ passes through the equator to complete the triad.[2]

2.1.2. ECEF frame

Earth Centered Earth Fixed frame (ECEF), like Earth Centered Inertial frame (ECI), is coincides with the center of mass of Earth. The difference of this frame is that the rotation of this frame around Earth’s rotation axis at Earth’s rotational speed. If the basis vectors of this axis set are determined as $\hat{\imath}_e\ \hat{\jmath}_e\ \hat{k}_e$. The direction of the $\hat{\imath}_e$ basis vector is located from the center of the Earth to the point where Greenwich meridian meets the equator. The $\hat{k}_e$ basis vector comes from the center of the Earth and is inverted with the spin axis of the world. The vector in $\hat{\jmath}_e$ passes from the equator and is the complementary vector of the Cartesian system.

A transformation can be made between ECI and ECEF using the Earth's rotational speed (ω_E). The angle between ECI and ECEF can be found with $\Omega = \omega_E(t - t_{J2000})$, where t is current time and t_{J2000} expresses J2000 epoch, while ω_E is equivalent to the angular velocity of the Earth. [2]

$$T_{ECI}^{ECEF} = \begin{bmatrix} \cos(\Omega) & \sin(\Omega) & 0 \\ -\sin(\Omega) & \cos(\Omega) & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad 2.1$$

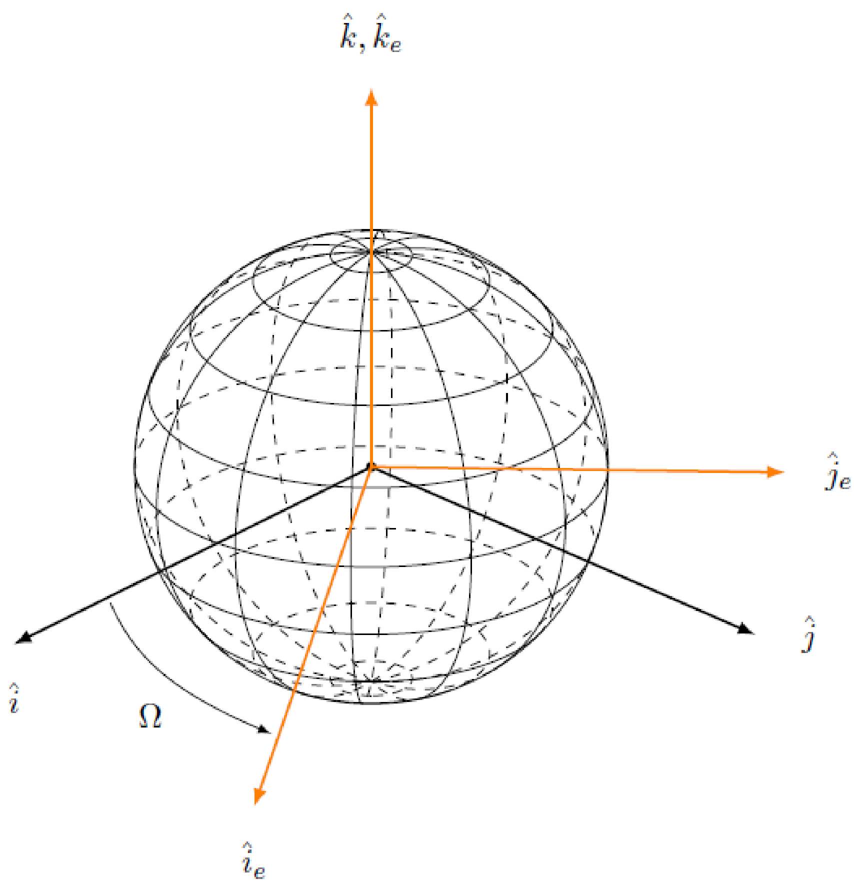

Figure 2.1.2.1:ECI and ECEF Frames adopted from Lustig [2017].

2.1.3. Body frame

Body frame is a commonly used frame for determining orientation on planes, satellites or rockets and obtaining equations of motion. The origin of this frame, which is translated and rotated with the rocket, is located in the center of gravity ($c.g$) of the vehicle or in other words in the center of mass ($c.m.$) of the rocket. If the body frames basis vectors are called $\hat{\imath}_b$ $\hat{\jmath}_b$ $\hat{k}_b$, the $\hat{\imath}_b$ vector is usually determined from the $c.m$ of the vehicle towards the nose. While the $\hat{\jmath}_b$ axis of the rocket is determined from $c.m.$ to the right fin, the $\hat{k}_b$ axis is determined from $c.m.$ downwards to complete the triad[2]. Body Frame can is shown in Figure 2.1.3.1.

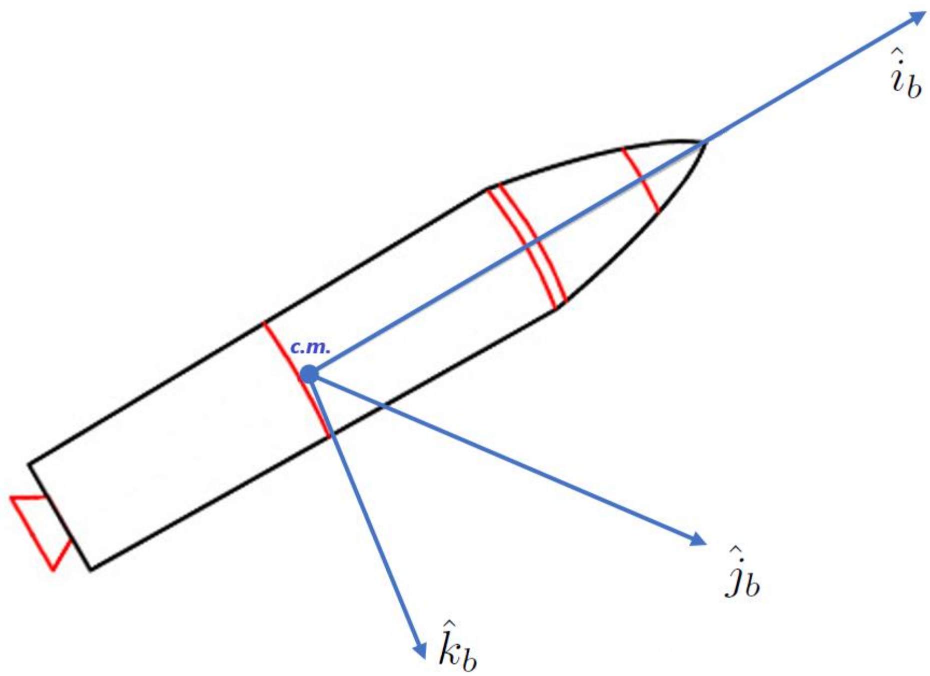

Figure 2.1.3.1: Body Frame.

When determining the attitude of the rocket at any moment, roll(ϕ), pitch(θ) and yaw(ψ) angles are used. These angles correspond to rotations around $\hat{\imath}_b$, $\hat{\jmath}_b$ and $\hat{k}_b$ basis vectors, respectively. The body frame transformation matrix from the inertial frame is as follows.

$$T_I^B = \begin{bmatrix} \cos(\theta)\cos(\psi) & \cos(\theta)\sin(\psi) & -\sin(\theta) \\ \cos(\psi)\sin(\theta)\sin(\phi)-\cos(\phi)\sin(\psi) & \cos(\phi)\cos(\psi)+\sin(\theta)\sin(\phi)\sin(\psi) & \cos(\theta)\sin(\phi) \\ \sin(\phi)\sin(\psi)+\cos(\phi)\cos(\psi)\sin(\theta) & \cos(\phi)\sin(\theta)\sin(\psi)-\cos(\psi)\sin(\phi) & \cos(\theta)\cos(\phi)) \end{bmatrix} \quad 2.2$$

2.1.4. Non-Rolling body frame

Non rolling Body frame is located between Inertial frame and Body frame. Origin of Non Rolling Body Frame is located at the $c.g.$ of the vehicle. It is obtained by two transformations to be yaw and pitch, respectively, from the Inertial frame. Thus, the secondary axis of the Non Rolling Frame remains in the horizontal plane. The primary axis is in line with the primary axis of the Body frame, and the third axis is in the vertical position. It is common to use in vehicles with rotational symmetry. It is used to separate the pitch and yaw channels with the roll channel since it does not have a roll motion as it can be understood from the Non Rolling Frame name. It is more convenient to use this frame to model the Magnus moments and forces that occur in spinning symmetrical bodies. [3]

The conversion between the non-rolling body frame and the body frame is done with the roll angle (ϕ) found by integrating the spin rate (ω).

$$T_{B'}^{B} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & \sin\phi \\ 0 & -\sin\phi & \cos\phi \end{bmatrix} \qquad 2.3$$

2.1.5. Wind and aeroballistic wind frame

The planes and missiles are exposed to a relative wind while moving in the air. This relative wind creates aerodynamic forces and moments. Since the forces and moments that are generated are defined in the wind reference frame, understanding of wind frame is important. The origin of wind frame is at the $c.m.$ of the vehicle. While the basis vector in i direction is parallel to the relative wind direction, the directions of other basis vectors vary according to the type of vehicle.

Transformation matrix between body frame and wind frame can be obtained using Cartesian alpha and beta incidence angles for aircrafts or rockets with planar symmetry or polar incidence angles which are total angle of attack and aerodynamic roll angles for rotationally symmetric rockets [3].

The frame obtained by rotating the body frame around the y axis with negative alpha angle is called stability frame. If the angle to be transformed is desired to be positive, the body frame is obtained by rotating the stability frame with positive alpha, the transformation matrix between stability and body frames is found as Equation 2.3.

$$T_S^B = \begin{bmatrix} \cos\alpha & 0 & -\sin\alpha \\ 0 & 1 & 0 \\ \sin\alpha & 0 & \cos\alpha \end{bmatrix} \tag{2.3}$$

Wind frame is obtained by rotating the stability frame in z axis with beta angle.

$$T_S^W = \begin{bmatrix} \cos\beta & \sin\beta & 0 \\ -\sin\beta & \cos\beta & 0 \\ 0 & 0 & 1 \end{bmatrix} \tag{2.4}$$

For vehicles with planar symmetry, the total transformation matrix from body frame to wind frame is shown in Equation 2.5.

$$\begin{gathered} T_B^W = T_S^W \overline{T_S^B} \\ T_B^W = \begin{bmatrix} \cos\alpha\cos\beta & \sin\beta & \sin\alpha\cos\beta \\ -\cos\alpha\sin\beta & \cos\beta & -\sin\alpha\sin\beta \\ -\sin\alpha & 0 & \cos\alpha \end{bmatrix} \end{gathered} \tag{2.5}$$

For vehicles with rotational symmetry, the aeroballistic frame is obtained by rotating the body axis to -phi ($\acute{\phi}$). The feature of this axis is that the basis vectors in i and k are located on the load factor plane.

$$T_A^B = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi' & \sin\phi' \\ 0 & -\sin\phi' & \cos\phi' \end{bmatrix} \tag{2.6}$$

In order to switch from aeroballistic frame to aeroballistic wind frame, the aeroballistic frame should be rotated with the total angle of attack. Thus, the aeroballistic wind frame, in which forces and moments are defined, is obtained as Equation 2.7.

$$T_W^A = \begin{bmatrix} \cos\alpha' & 0 & -\sin\alpha' \\ 0 & 1 & 0 \\ \sin\alpha' & 0 & \cos\alpha' \end{bmatrix} \tag{2.7}$$

The conversion matrix that is formed by multiplying these two matrices is the matrix that will give aeroballistic wind frame conversion from body frame.

$$T_B^W = \bar{T}_W^B = \bar{T}_W^A \bar{T}_A^B$$
$$T_B^W = \begin{bmatrix} \cos\alpha' & \sin\alpha'\sin\phi' & \sin\alpha'\cos\phi' \\ 0 & \cos\phi' & -\sin\phi' \\ -\sin\alpha' & \cos\alpha'\sin\phi' & \cos\alpha'\cos\phi' \end{bmatrix} \qquad 2.8$$

2.2. Rotational Kinematics

Since we are dealing with different frames transformation between them are important. Roll, Pitch and Yaw angles are commonly used attitude angles to represent vehicles orientation with respect to the inertial frame and can be used for transformation between frames.

An angular velocity is used which is derivative of yaw pitch and roll angles can be used to obtain these angles [4].

Since

$$\vec{\omega} = p\hat{\imath}_b + q\hat{\jmath}_b + r\hat{k}_b \qquad 2.9$$

We can represent the angular velocity as shown below

$$\vec{\omega} = \begin{bmatrix} p \\ q \\ r \end{bmatrix}$$

$$= \begin{bmatrix} \dot{\phi} \\ 0 \\ 0 \end{bmatrix} + R(x,\phi)\begin{bmatrix} 0 \\ \dot{\theta} \\ 0 \end{bmatrix} + R(x,\phi)R(y,\theta)\begin{bmatrix} 0 \\ 0 \\ \dot{\psi} \end{bmatrix}$$

$$= \begin{bmatrix} \dot{\phi} - \dot{\psi}\sin\theta \\ \dot{\theta}\cos\phi + \dot{\psi}\cos\theta\sin\phi \\ \dot{\psi}\cos\theta\cos\phi - \dot{\theta}\sin\phi \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & -\sin\theta \\ 0 & \cos\phi & \sin\phi\cos\theta \\ 0 & -\sin\phi & \cos\phi\cos\theta \end{bmatrix} \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} \qquad 2.10$$

And one step after we get Equation 2.11.

$$\begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix} = \frac{1}{\cos\theta} \begin{bmatrix} \cos\theta & \sin\phi\sin\theta & \cos\phi\sin\theta \\ 0 & \cos\phi\cos\theta & -\sin\phi\cos\theta \\ 0 & \sin\phi & \cos\phi \end{bmatrix} \begin{bmatrix} p \\ q \\ r \end{bmatrix} \tag{2.11}$$

The drawback of the usage of the roll, pitch and yaw angles becomes clearly visible in this equation. When the pitch angle becomes $\pi/2 + k\pi$ where $k \in Z$ singularities appear. These singularities are commonly seen in gyroscopic application and high pitch angle maneuver for aircrafts or missiles and called as gimbal lock. What it does to the system is it makes yaw and roll angle undistinguishable when gimbal lock occurs. To prevent these problem, we need to express the attitude angles in another way. At this point, Hamilton's quaternions are used. Quaternions are 4-dimensional numbers of which 1 is real and 3 is imaginary [5].

We can express the quaternions as follows.

$$q = \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} \tag{2.12}$$

In addition, there is a connection between the quaternions' rate of change and the angular velocities of the vehicle as follows.

$$\begin{pmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 & -p & -q & -r \\ p & 0 & r & -q \\ q & -r & 0 & p \\ r & q & -p & 0 \end{pmatrix} \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} \tag{2.13}$$

The quaternion values here represent attitude. But in order for this process to be valid, the norm of the quaternions must be equal to 1. For this, the expression $k \times \lambda \times q$ is added to the right side of the equation, so that the norm is kept at 1.

Where, $\lambda = 1 - (q_0^2 + q_1^2 + q_2^2 + q_3^2)$ and $k\Delta t \leq 1(\ \Delta t\ integration\ interval)$

In order to start the process above, the first values of the quaternions must be known. Instead of dreaming or memorizing, since quaternions have no meaning, the first values of quaternions can be calculated by using the roll pitch and yaw angles with the help of the following equation [3].

$$
\begin{aligned}
q_0 &= \cos\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right)+\sin\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right)\\
q_1 &= \cos\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right)-\sin\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right)\\
q_2 &= \cos\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right)+\sin\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right)\\
q_3 &= \sin\left(\frac{\psi}{2}\right)\cos\left(\frac{\theta}{2}\right)\cos\left(\frac{\phi}{2}\right)-\cos\left(\frac{\psi}{2}\right)\sin\left(\frac{\theta}{2}\right)\sin\left(\frac{\phi}{2}\right)
\end{aligned}
\tag{2.14}
$$

In addition to eliminating the singularities coming with the Euler angles, the quaternions provide ease in mathematical operations since they are composed of scalar numbers [5].

2.3. Translational Dynamics

Translational dynamics are obtained from Newton's second law with respect to the inertial frame I which states that the change in the vehicles linear momentum is equal to the externally applied forces[3].

$$mD^I\boldsymbol{v}_B^I = f_{a,p} + mg \tag{2.15}$$

Since the Earth is assumed to be inertial frame Equation 2.15 becomes,

$$mD^E\boldsymbol{v}_B^E = f_{a,p} + mg \tag{2.16}$$

But it is desired to obtain the equations in the body frame so Euler transformation should be applied to the Equation 2.16.

$$mD^B\boldsymbol{v}_B^E + m\boldsymbol{\Omega}^{BE}\boldsymbol{v}_B^E = \boldsymbol{f}_{a,p} + m\boldsymbol{g} \tag{2.17}$$

Where the skew symmetric matrix $\boldsymbol{\Omega}^{BE}$ represents the rotation of Body frame with respect to the Earth frame [3].

If we re arrange the Equation 2.17 that left hand side is time rate of change of velocity, we obtain the translational equations of motion.

$$m\left\{\begin{bmatrix}\frac{\mathrm{d}u}{\mathrm{d}t}\\ \frac{\mathrm{d}v}{\mathrm{d}t}\\ \frac{\mathrm{d}w}{\mathrm{d}t}\end{bmatrix}^B + \begin{bmatrix}0 & -r & q\\ r & 0 & -p\\ -q & p & 0\end{bmatrix}^B \begin{bmatrix}u\\ v\\ w\end{bmatrix}^B\right\} = \begin{bmatrix}f_{a,p_1}\\ f_{a,p_2}\\ f_{a,p_3}\end{bmatrix}^B + \begin{bmatrix}t_{11} & t_{12} & t_{13}\\ t_{21} & t_{22} & t_{23}\\ t_{31} & t_{32} & t_{33}\end{bmatrix}^{BL}\begin{bmatrix}0\\ 0\\ mg\end{bmatrix}^L \tag{2.18}$$

Scalar form of the Equation 2.18 can be expressed as,

$$\begin{aligned} \frac{\mathrm{d}u}{\mathrm{d}t} &= rv - qw + \frac{f_{a,p_1}}{m} + t_{13}g \\ \frac{\mathrm{d}v}{\mathrm{d}t} &= pw - ru + \frac{f_{a,p_2}}{m} + t_{23}g \\ \frac{\mathrm{d}w}{\mathrm{d}t} &= qu - pv + \frac{f_{a,p_3}}{m} + t_{33}g \end{aligned} \qquad 2.19$$

Once the integration of these differential equations shown in Equation 2.19 are completed it gives the velocity vector. In addition to obtain the location of the center of mass of the vehicle with respect to the Earth reference point E the velocity vector should be integrated once more.

$$[D^E s_{BE}] = [v_B^E] \qquad 2.20$$

Where, s_{BE} is the location of the center of mass of the vehicle with respect to the Earth reference point E. Since the change of the location of the $c.m.$ is necessary to calculate gravity vector it is calculated in Earth frame.

2.4. Rotational Dynamics

The rotational Dynamics are obtained with the help of the Euler's second law which states that the externally applied moments are equal to the time rate of change of the angular momentum of the vehicle [3].

$$D^E l_B^{BE} = m_B \qquad 2.21$$

Where l_B^{BE} is angular momentum and equals to $I_B^B \omega^{BE}$ where I_B^B is moment of inertia of the missile body expressed in body coordinates system and ω^{BE} is the rotation of missile body with respect to the Earth frame.

Since the derivative in the equation of motion is takes place in the earth frame the Euler transform should be applied just like in translational dynamics to obtain the equations of motion in the body frame.

$$D^B l_B^{BE} + \Omega^{BE} l_B^{BE} = m_B \qquad 2.22$$

First term in the left hand side is the rotational derivative of angular momentum. If we expand the angular momentum vector and apply the chain rule the Equation 2.22 becomes,

$$D^B l_B^{BE} = D^B(I_B^B \omega^{BE}) = D^B I_B^B \omega^{BE} + I_B^B D^B \omega^{BE} = I_B^B D^B \omega^{BE} \qquad 2.23$$

Since rigid body assumption takes place $D^B I_B^B$ is zero and Equation 2.22 becomes

$$\boldsymbol{I}_B^B D^B \boldsymbol{\omega}^{BE} + \boldsymbol{\Omega}^{BE} \boldsymbol{I}_B^B \boldsymbol{\omega}^{BE} = \boldsymbol{m}_B \qquad 2.24$$

First term on the left hand side is the angular momentum change due to time rate of change in the angular velocity of the body frame with respect to the Earth frame, second term is the gyroscopic effect which is occurs due to choosing Body frame as a reference frame [3].

If we re arrange the Equation 2.24 by multiplying from left side of the both equations with the inverse of the moment of inertia tensor and the equations become,

$$\left[\frac{\mathrm{d}\omega^{BE}}{\mathrm{d}t}\right]^B = ([I_B^B]^B)^{-1}(-[\Omega^{BE}]^B [I_B^B]^B [\omega^{BE}]^B + [m_B]^B) \qquad 2.25$$

For a rocket with a diagonal moment of inertia tensor, Equation 2.25 can be written in scalar form as follows,

$$\begin{aligned} \frac{\mathrm{d}p}{\mathrm{d}t} &= I_1^{-1}\left[(I_2 - I_3)qr + m_{B_1}\right] \\ \frac{\mathrm{d}q}{\mathrm{d}t} &= I_2^{-1}[(I_3 - I_1)pr + m_{B2}] \\ \frac{\mathrm{d}r}{\mathrm{d}t} &= I_3^{-1}\left[(I_1 - I_2)pq + m_{B_3}\right] \end{aligned} \qquad 2.26$$

2.5. Subsystem Models

2.5.1. Atmosphere

The atmospheric model should be included in the mathematical modeling of the rocket, since the atmospheric properties have a great importance on aerodynamic forces and moments. In the rocket flight regime, atmospheric features such as pressure, intensity, sound velocity and temperature need to be modeled. Although US Standard Atmosphere

1976 provides the necessary model to be created in this regard, there is no need for such a detailed atmosphere model within the scope of this thesis.

In this section, formulas expressing the characteristics of the atmospheric features in different layers of the atmosphere will be given by using US Standard Atmosphere 1976 with some assumptions.

2.5.1.1. Temperature

The temperature has different rates of change in 5 different regions of atmosphere which are 0km-86km, 86km-91km, 91km-110km, 110km-120km and finally 120-1000 km.

For the temperature change, the Equation 2.27 can be used up to 86km altitude [6].

$$T = T_{0,b} + L_{M,b} \cdot (H - H_b) \qquad 2.27$$

Where T is the temperature at the desired altitude, $T_{0,b}$ is the temperature in the previous layer, H_b is altitude where temperature gradient changes, $L_{M,b}$ is temperature gradient and H is instant altitude. The values of these constants are given in the table 2.5.1.1.1. [6]

b	Geopotential height H_b (km)	Temperature gradient $L_{M,b}$ (K/km)
0	0	-6.5
1	11	0
2	20	1
3	32	2.8
4	47	0
5	51	-2.8
6	71	-2
7	84.852	-

Table 2.5.1.1.1: Temperature calculation table for altitude between 0-85 km

The temperature between 86 km and 91 km is constant and has a value of T = 186.87K.

The temperature can be calculated between 91km and 110 km with the help of the Equation 2.28.

$$T = 263.1905 - 76.3232\left(1 - \left(\frac{H - 91}{-19.9429}\right)^2\right)^{0.5} \tag{2.28}$$

Where H is the altitude limited to the range of 91 to 110 km.

For altitude 110 km to 120 km there is a linear relationship between the altitude and temperature.

$$T = 240 + 12(H - 110) \tag{2.29}$$

For 120 to 1000 km there is an exponential relationship between temperature and altitude.

$$T = T_\infty - (T_\infty - 360)\exp(-\lambda\xi) \tag{2.30}$$

Where, $$T_\infty = 1000km$$

$$\lambda = \frac{12}{T_\infty - 360} = 0.01875$$

$$\xi = (h - 120)\left(\frac{r_0 + 120}{r_0 + h}\right) \tag{2.31}$$

$$r_0 = 6.356766 \times 10^6 \text{km.}$$

2.5.1.2. Pressure

In the standard atmosphere model, two pressure models are proposed for altitudes between 0 km and 86 km, and one for 86 km and 1000 km. Although Equation 2.32 is sufficient for the model we will create. [6]

$$P = P_b\left(1 - \frac{LH}{T_0}\right)^{\frac{gM}{RL}} \tag{2.32}$$

Where, $g\ [\mathrm{m/s^2}]$ is gravitational acceleration, $M = 0.028964\ \mathrm{kgmol^{-1}}$ is molar mass of air, $R = 8{:}31447\ \mathrm{J(Kmol)^{-1}}$ is universal gas constant. $P_b = 101325\ Pa$ is Sea level standard atmospheric pressure and $L = 0.0065\ Km^{-1}$ is temperature gradient.

2.5.1.3. Density

Density can be calculated by using the ideal gas law with the calculated temperature and pressure values.

$$\rho = \frac{pM}{RT} \qquad 2.33$$

2.5.1.4. Speed of sound

$$c = \left(\frac{\gamma RT}{M_0}\right)^{0.5} \qquad 2.34$$

where $\gamma = 1.4$ is the ratio between specific heat at constant pressure and constant volume. R is the gas constant M is the molar mass of the gas and T is the temperature.

2.5.2. Gravity model

WGS 84 includes the gravitation model of the world in detail. This model is in the form of scalar potential function(V). The potential function used in WGS-84 is Earth Gravitational Model 1996 (EGM96). Although, it has 130,676 coefficients the largest coefficient is two orders of magnitude bigger than the next coefficient. If we neglect the other coefficients the result is very accurate. Besides, it removes the dependence on terrestrial longitude. [4] The potential function $\left(V(r,\psi)\right)$ at a point $P(r,\psi)$ is shown in Equation 2.35.

$$V(r,\psi) = \frac{GM}{r}\left[1 - \frac{1}{2}\left(\frac{a}{r}\right)^2 J_2(3\sin^2\psi - 1)\right] \qquad 2.35$$

Where, ψ is the geocentric latitude r is the length of the position vector. The Earth's gravitational constant is given as $GM = 398\ 6004.418\ \times\ 10^8 \mathrm{m}^3\ /\ \mathrm{s}^2$. J_2 is constant given by the Equation 2.36.

$$J_2 = -\sqrt{5}C_{2,0}\ = 1.082626684 \times 10^{-3} \qquad 2.36$$

Gravitational acceleration can represent by vector

$$g = g_x\hat{\imath} + g_y\hat{\jmath} + g_z\hat{k} \qquad 2.37$$

Then we obtain the following individual components of gravitational acceleration

$$G_x = -\frac{GM}{r^2}\left[1 + \frac{3}{2}J_2\left(\frac{a}{r}\right)^2 [1 - 5\sin^2\psi]\right]\frac{x}{r}$$

$$G_y = -\frac{GM}{r^2}\left[1 + \frac{3}{2}J_2\left(\frac{a}{r}\right)^2 [1 - 5\sin^2\psi]\right]\frac{y}{r} \qquad 2.38$$

$$G_z = -\frac{GM}{r^2}\left[1 + \frac{3}{2}J_2\left(\frac{a}{r}\right)^2 [3 - 5\sin^2\psi]\right]\frac{z}{r}$$

Where x,y and z are the ECEF components and r is the length of geocentric position vector p. In order to obtain gravity vector $\boldsymbol{g}$**,** centripetal acceleration is subtracted from gravitational acceleration [4]. Then, gravity vector is obtained as,

$$g = \mathrm{G} - \omega_{e/i} \times \left(\omega_{e/i} \times \mathrm{p}\right) \qquad 2.39$$

Where, $\omega_{e/i}$ is Earth's (constant) inertial angular velocity. If the Earth frame is chosen as an internal frame, the centripetal acceleration expression arising from the angular rotation of the world disappears.

2.5.3. Aerodynamics

In order to complete translational and rotational equations, aerodynamic forces and moments which are caused by the relative air flow over the rocket should be calculated. Aerodynamic forces and moments are calculated using the obtained aerodynamic coefficients from Missile DATCOM [7].

Aerodynamic forces can be written as follows:

$$\mathrm{F}_A = QS\begin{bmatrix} C_X \\ C_Y \\ C_Z \end{bmatrix} \qquad 2.40$$

Here, Q is dynamic pressure S is reference area, while C_X, C_Y and C_Z represent the force coefficients in the body frame.

The aerodynamic coefficients depend on Mach number, total angle of attack, aerodynamıc roll angle phi and angular rates. They do not depend on control surfaces deflections in this model. These coefficients need to be determined with these dependencies. Equation 2.41 shows the total angle of attack and aerodynamic roll angle formulation. [3]

$$\alpha' = cos^{-1}\left(\frac{u}{V}\right)$$

$$\phi' = tan^{-1}\left(\frac{u}{w}\right) \qquad 2.41$$

Where, α' and ϕ' is total angle of attack and aerodynamic roll angle respectively. u, w and V represent the velocities.

Then, the force equation can be defined as shown in Equation 2.42.

$$\mathrm{F}_A = QS\begin{bmatrix} -C_A + C_{A_q} q \frac{d}{2V_\infty} \\ C_Y + C_{Y_q} q \frac{d}{2V_\infty} \\ -C_N + C_{N_q} q \frac{d}{2V_\infty} \end{bmatrix} \qquad 2.42$$

Moments can be calculated in a similar manner. Since the missiles fuel mass is decreasing during engine burn, missiles $c.g$ is changing. Therefore, aerodynamic moments must be moved to the new center of gravity. Also, since the reference center of gravity in Missile DATCOM is the nose of the rocket, the difference between the new center of gravity and the reference center of gravity should be considered. [8]

$$M_A = QSd\begin{bmatrix} C_l + C_{\mathrm{l}} q \frac{d}{2V_\infty} + C_{l_p} p \frac{d}{2V_\infty} + C_{l_r} r \frac{d}{2V_\infty} \\ C_m + \left(C_{m_q} + C_{m_{\dot\alpha}}\right) q \frac{d}{2V_\infty} + C_{m_p} p \frac{d}{2V_\infty} + C_{m_r} r \frac{d}{2V_\infty} \\ C_n + C_{\mathrm{n}} q \frac{d}{2V_\infty} + C_{n_p} p \frac{d}{2V_\infty} + C_{n_r} r \frac{d}{2V_\infty} \end{bmatrix} + \overline{X}_{cg} \times \overline{F}_A \qquad 2.43$$

The input file used to obtain data from Missile DATCOM is given in Appendix.

2.5.4. Thrust

Aerobee 150A has a single thrust source. The purpose of this engine is to take the rocket to a certain altitude. Since it is a fixed nozzle, the engine produces thrust only in axial direction and does not create any moment since it does not have a force arm.

135 kg of fuel in the Aerobee 150A rocket burns with constant flow rate for 51 seconds and produces constant 18KN force [9]. Thrust force created by the engine in the body frame can be written as shown in Equation 2.44.

$$\mathrm{F}_T = \begin{bmatrix} 18\ \mathrm{kN} \\ 0 \\ 0 \end{bmatrix} \tag{2.44}$$

3.SIMULATION

Rotational and translational equations that define the movement of the rocket are obtained. The model to be simulated will be created by establishing the relations of these equations with environmental factors in the computer environments called MATLAB and SIMULINK.

Before combining dynamic equations with environmental factors, the relation of moment of inertia should be taken into consideration. It is known that the Moment of Inertia in the dynamic equations have changed as the rocket's fuel decreases over time. Therefore, instant Moment of Inertia must be calculated.

The rocket's center of gravity, weight and moment of inertia values are given in the table 3.1. These values are the values of the rocket without oxidizer and fuel[9].

Without Fuel and Oxidizer	Value
Xcg Empty (m)	3.596
Empty Mass (kg)	221.404
Ixx (kgm2)	7.1452
Iyy (kgm2)	1095.6903
Izz (kgm2)	1095.6903

Table 3.1: Empty rocket parameters

To calculate the instantaneous moment of inertia of the rocket, we need to calculate the total center of gravity of the rocket. For this, the weight and center of gravity information of the empty state of the rocket, fuel and oxidizer are required. The total center of gravity of the rocket can be found with the Equation 3.1.

$$TCG = \frac{(RM * RCG) + (FM * FCG) + (OM * OCG)}{RM + FM + OM} \qquad 3.1$$

Where; TCG is total center of gravity position, RCG is empty rockets center of gravity location, FCG is location of center of gravity of fuel, OCG is oxidizers center of gravity location, RM is rockets empty mass, FM is fuel mass, OM is oxidizer mass.

According to the parallel axis theorem, the moment of inertia in the X, Y and Z axes are found with the Equation 3.2.

$$TI_{xx} = RI_{xx} + FI_{xx} + OI_{xx} \qquad 3.2$$

Where, RI_{xx} , FI_{xx} , OI_{xx} and TI_{xx} are empty rockets, fuels, oxidizers and total moments of inertias in Body frame's x axis respectively.

$$TI_{yy} = TI_{zz} = \left(RI_{yy} + RM(TCG - RCG)^2\right) + \left(FI_{yy} + FM(TCG - FCG)^2\right) + \left(OI_{yy} + OM(TCG - OCG)^2\right) \qquad 3.3$$

Where, I_{yy} indicates moment of inertia in Y axis and I_{zz} indicates moment of inertia in Z axis.

Here, since the fuel and oxidizer of the rocket are stored in solid cylinder tanks, Equation 3.4 and Equation 3.5 should be used in moment of inertia calculations.

$$I_{xx} = \frac{mr^2}{2} \tag{3.4}$$

$$I_{yy} = I_{zz} = \frac{m(3r^2 + h^2)}{12} \tag{3.5}$$

Where, m is the mass, r is the radius of the tank and h is the height of the tank.

3.1. Matlab Model

The physical parameters of the rocket are written in a Matlab script [9]. Since the model will be built on Simulink, all the necessary physical parameters throughout the model have been collected in a single script. The physical parameters in the Matlab file are shown in the Table 3.1.1.

PARAMETER	VALUE
Rocket Length (m)	7.34
Rocket Diameter (m)	0.381
Rocket Reference Area (m^2)	0.114
Nosecone Length (m)	2.23
Fin Root Chord (m)	1.0414
Fin Tip Chord (m)	0.6604
Fin Span (m)	0.673
Fin Sweep (deg)	45
Fin Distance (m)	6.2986
Canard Root Chord (m)	0.3
Canard Tip Chord (m)	0
Canard Span (m)	0.3
Canard Sweep (deg)	45
Canard Distance (m)	1.83
Empty Mass (kg)	221.404
Fuel Mass (kg)	135
Oxidizer Mass (kg)	344
Payload (kg)	90
Xcg Empty (m)	3.596
Max Xcg Fuel (m)	5.7386
Max Xcg Oxidizer (m)	6.8574
Oxidizer Tank Radius (m)	0.09525
Oxidizer Tank Height (m)	1.1938
Fuel Tank Radius (m)	0.09525
Fuel Tank Height (m)	2.032

Table 3.1.1: Physical Parameters of Rocket.

A similar method was followed to transfer the aerodynamic coefficient from the Missile DATCOM's output file to the Matlab environment.

The outputs of the Missile DATCOM program are read with a written Matlab script and the required coefficient is saved in the look up tables. These look up tables are 3 dimensional. Look up tables take values according to alpha and phi which are polar incidence angles in addition to the Mach number.

3.2. Simulink Model

The model is created on the Simulink platform by using the rocket's physical parameters, equations of motion and look up tables obtained from DATCOM and transferred to the Matlab ecosystem. Actuator dynamics have been neglected. Simulink blocks were used for environmental factors such as atmosphere and gravity. Due to its high reliability, Simulink blocks in Aerospace Toolbox have been used where possible. Necessary functions in the model are written as scripts as matfunction and added to the model.

Simulink model, in which the body frame is taken as a reference frame, consists of 4 main blocks, including translational equations, rotational equations, attitude calculation and displacement calculation. These four main blocks are the parts of the model that contain the related equations. they are linked to other blocks that provide the necessary inputs. The remaining blocks consist of, Aerodynamic block is the block where the look up tables are calculated according to the inputs and the forces and moments are calculated, environment where atmospheric properties and gravity is calculated, thrust and gravity force calculation block, finally inertia and mass block where related calculations made.

The screenshot showing the SIMULINK model and the related blocks is shown in figure 3.2.1.

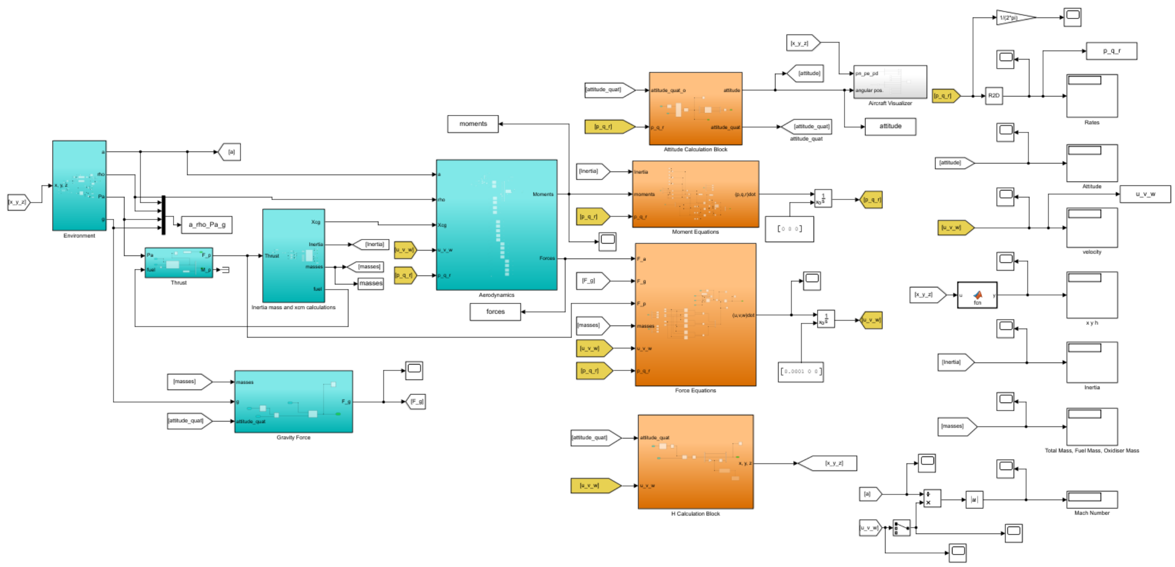

Figure 3.2.1: Simulink Model.

The orange blocks shown here are the main blocks and the cyan blocks are the remaining complementary blocks. The interior of the main and remaining complementary blocks that make up the model will be shown in appendix. As the rocket

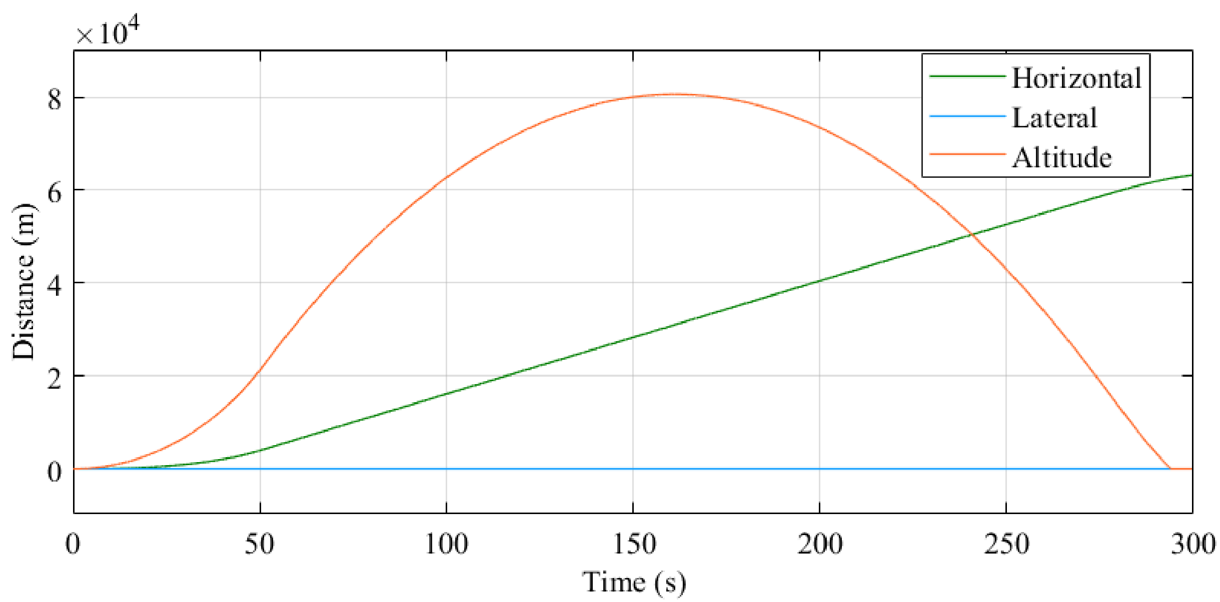

Figure 3.2.2: Distance travelled in each axis for 0 spin rate Body frame model.

is launched at 88 degrees from the ramp [10], necessary outputs are as follows.

The reason why the rocket travels along the horizontal axis is that the pitch of the rocket, which is 88 degrees on the ramp, decreases over time and the rocket has a horizontal speed. Horizontal distance increases parabolically until fuel runs out and continues to increase linearly after fuel runs out. Known as the peak of the altitude, the apogee is around 80 km. The vehicle does not displace on the lateral axis shown in Figure 3.2.2.

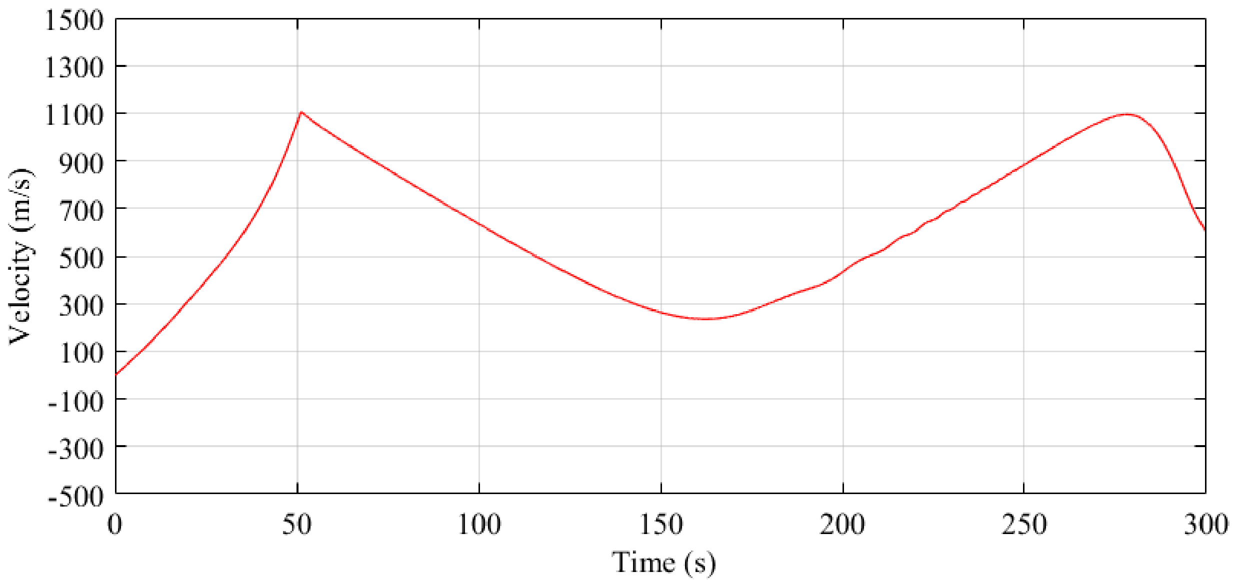

Figure 3.2.3: Velocity distribution for 0 Hz spin rate body frame model.

Figure 3.2.3 shows that the point where the velocity profile peaks in 51st second is the point where the fuel runs out and the speed is at 3.72 Mach level. Afterwards, the speed is decreasing and the minimum speed is reached in the apogee. Speed starts to increase again during descent. Due to the increasing air density after this time, the speed of the rocket decreases as the limit speed decreases continuously.

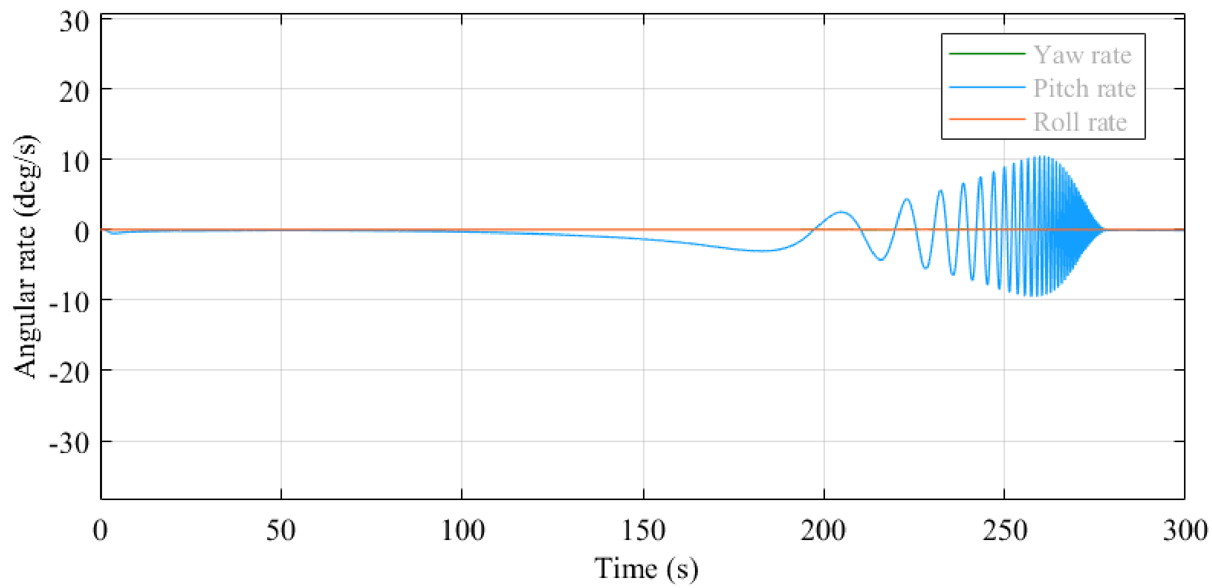

Figure 3.2.4: Angular rate over time for 0Hz spin rate body frame model.

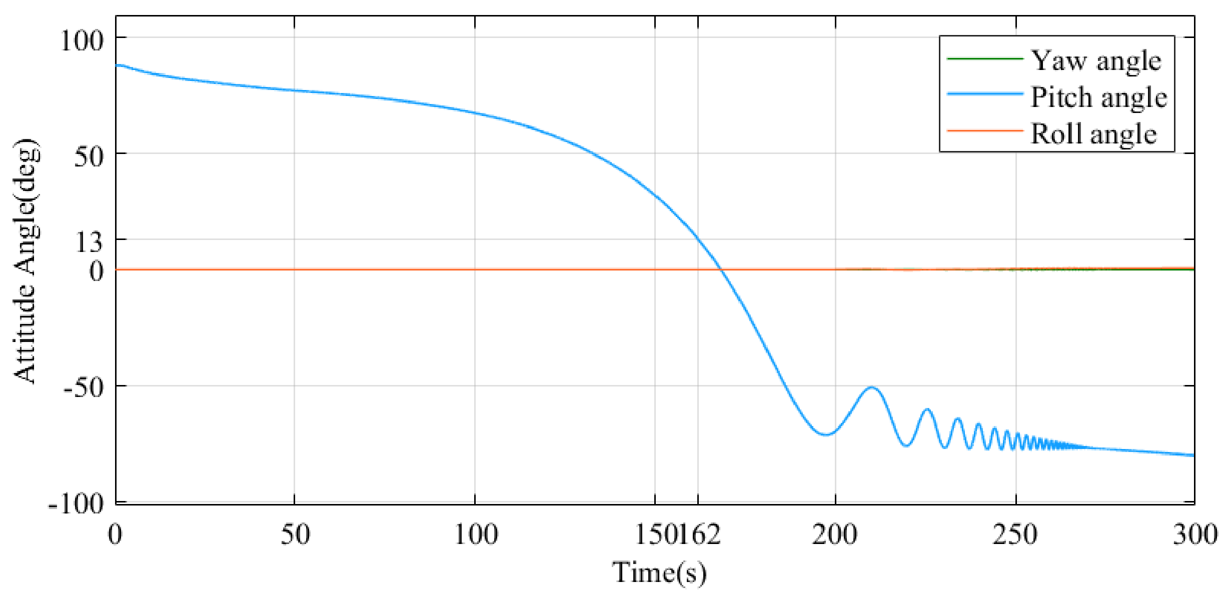

Figure 3.2.5: Attitude Angles for 0 Hz spin rate body frame model.

Attitude angles of this situation are shown in Figure 3.2.5. After the rocket is thrown from the ramp, theta angle (pitch angle) decreases continuously. The theta angle, which was 13 degrees while passing through the apogee at 162th second, fluctuated sinusoidally after 190 seconds and the rocket touched the ground with -80 degrees. The reason for the Roll and yaw angles, known as the phi and psi angles of the rocket, does not change, is that there is not any control input given to the vehicle and there is no defect in the air to create this effect.

In order to prevent fluctuation in the speed profile of the rocket after 190 seconds, to prevent the rocket from fluctuating at the pitch angle and to increase the stability of the rocket, a -1 Hz disturbance will be given to the rocket. Thus, the rocket will rotate with a spin rate of 1 Hz, as it is stable in the roll channel. The output of the rocket rotating with 1 Hz is as follows.

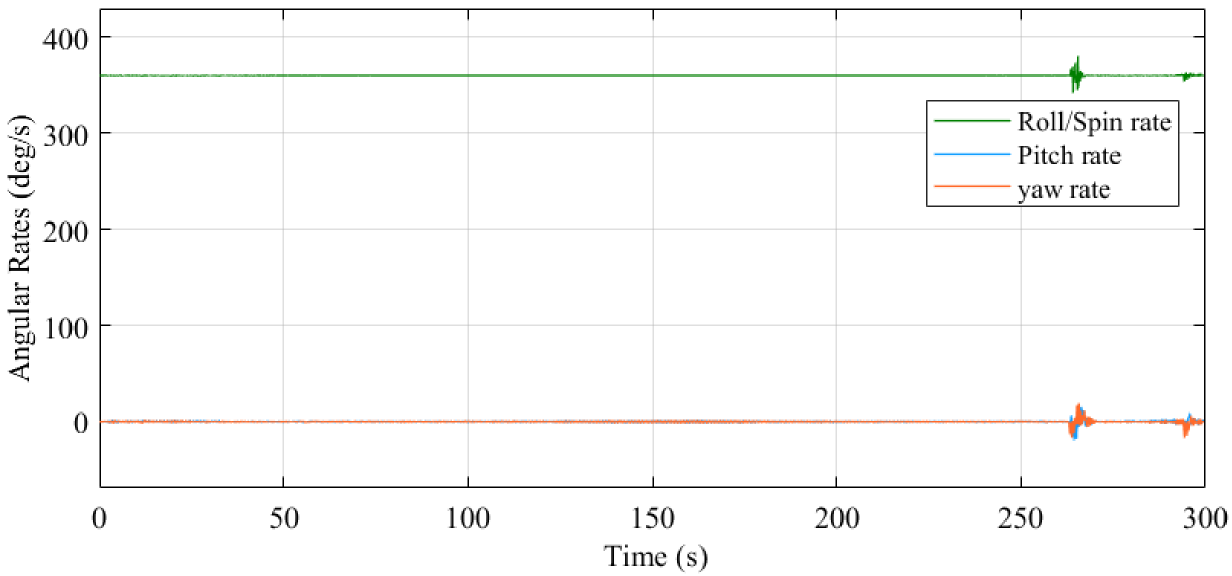

Figure 3.2.6: Angular rates for 1 Hz spin rate body frame model.

As we can see in Figure 3.2.6, the rocket rotates with 1 Hz for a given -1 Hz disturbance, and there are no angular rates on other axes. The cause of the sinusoidal rates at the end of the flight is not known and is thought to occur as a result of cross couplings.

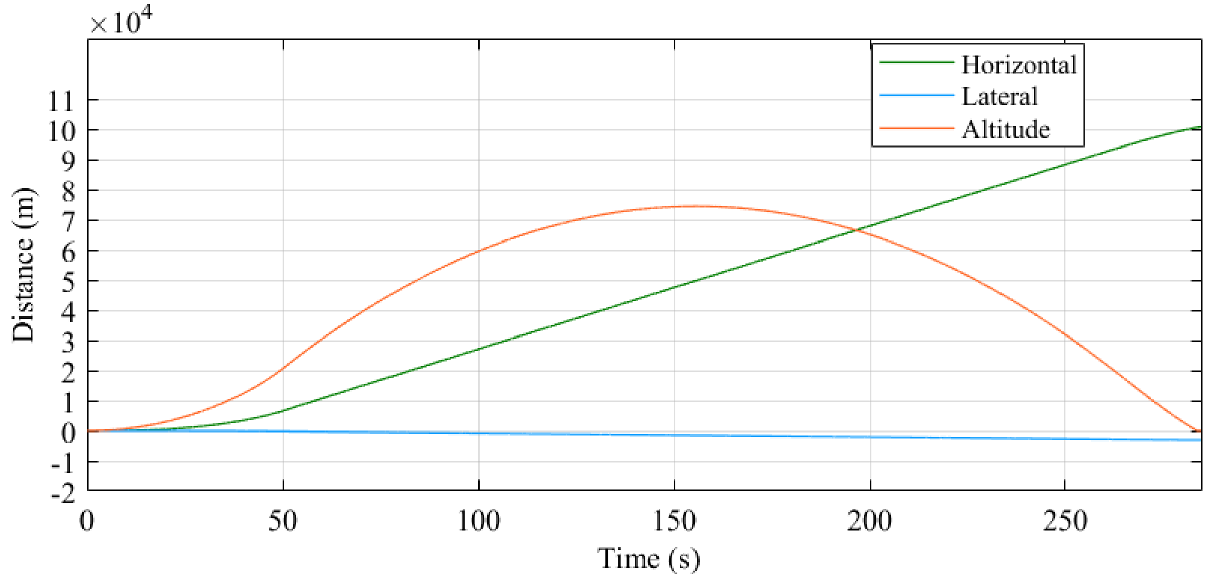

Figure 3.2.7: Distance travelled in each axes for 1 Hz spin rate body frame model.

In the flight, where the rocket rotated at 1 Hz, the maximum altitude decreased by 6 km to 75.6 km. It is observed that the distance traveled by the vehicle in the horizontal direction increases and rocket traveled small distance in the lateral axis.

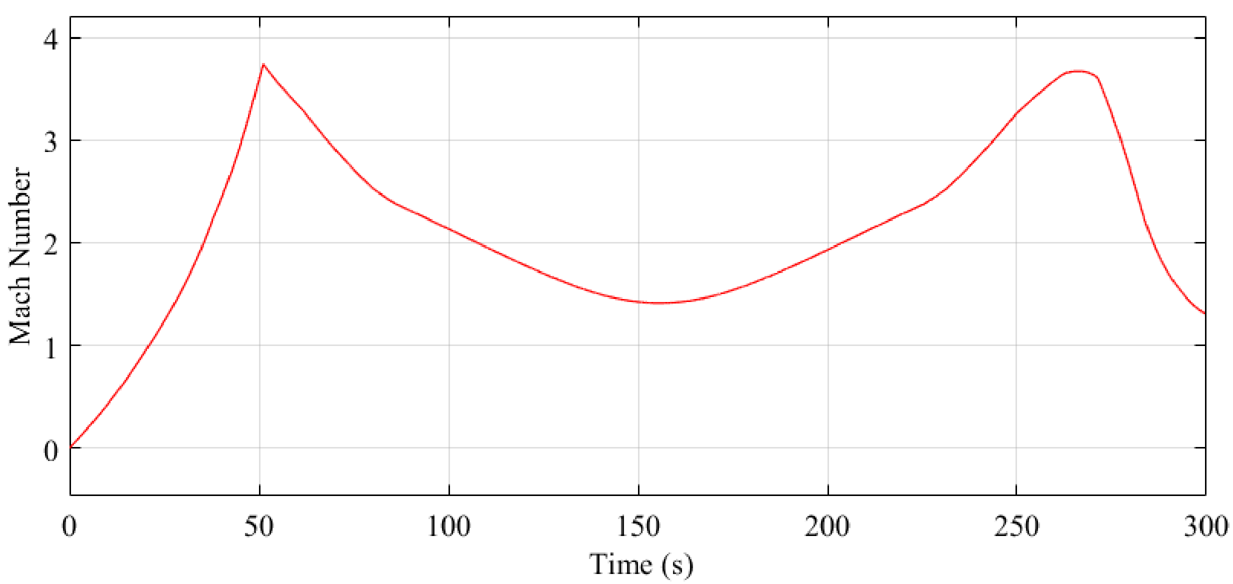

Figure 3.2.8: Mach number over time for 1 Hz spin rate body frame model.

The fluctuations in the speed profile, which is observed in non-rotating vehicles flight, disappeared and the speed of the vehicle in 51st second is similar to the previous one with 3.74 mach.

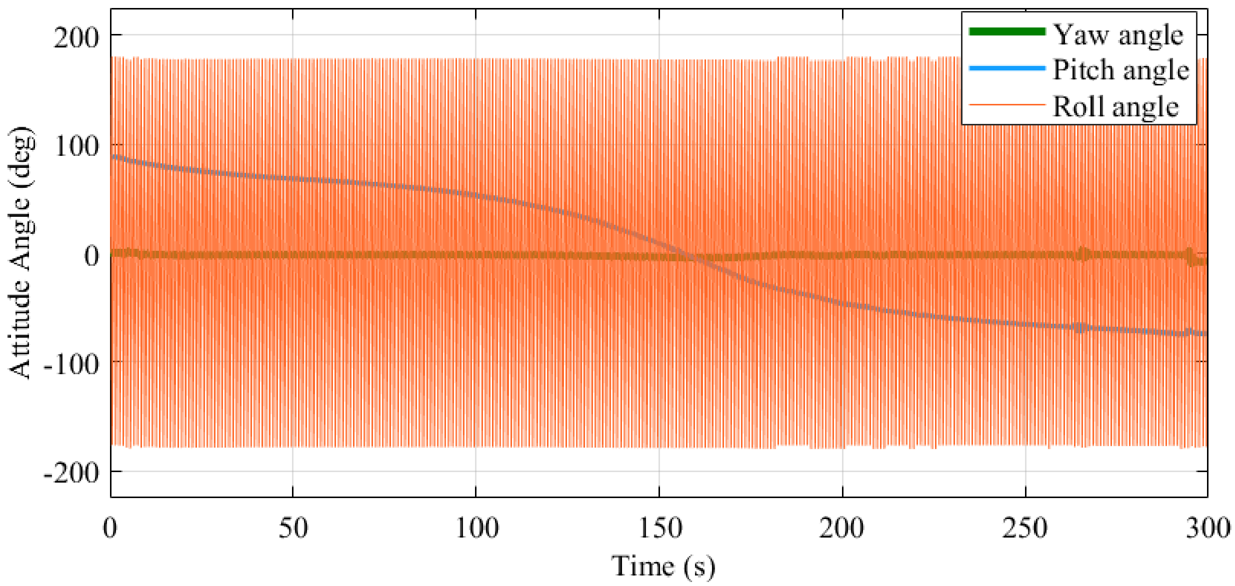

Figure 3.2.9: Attitude angles for 1 Hz spin rate body frame model.

The sinusoidal movement at the value of the pitch angle of the rocket after the apogee is sufficiently damped. Since the vehicle has a fixed roll rate, the roll angle changes continuously between $-\pi/2$and $+\pi/2$. Minor changes were observed in the yaw angle.

Although dynamic equations of the rotating rockets with the fixed spin rate seen above can be written in body frame, the body frame is not suitable for the spinning rocket's dynamics. If the rocket rotates with a fixed roll rate, the Non-Rolling Body frame will be a more accurate choice.

Dynamic equations written in the body frame will be transferred to the Non-Rolling Body (Body prime) frame because it is more suitable for the spinning rocket. For this reason, dynamic equations written in the body frame will be transferred to the Non-Rolling Body frame.

Translational equations written in the Body reference frame (B) can be transferred to the Non-Rolling Body frame (B') since translational equations of motions are valid in all frames[3]. The only change that needs to be made is to replace B with B' in equation 2.17.

$$mD^{B'}\boldsymbol{v}_{B'}^{E} + m\boldsymbol{\Omega}^{B'E}\boldsymbol{v}_{B'}^{E} = \boldsymbol{f}_{a,p} + m\boldsymbol{g} \quad \mathbf{3.6}$$

Transferring rotational equations are more difficult than translational equations. If we take the reference frame as body prime in Equation 2.22**.**

$$D^{B'} l_B^{BE} + \Omega^{B'E} l_B^{BE} = m_B \tag{3.7}$$

If we divide the angular velocity into two parts as the body primes angular velocity according to the inertial frame and the body frames angular velocity according to the body prime [3],

$$\omega^{BE} = \omega^{BB'} + \omega^{B'E} \tag{3.8}$$

Thus, l_B^{BE} can be written as below.

$$l_B^{BE} = I_B^B \omega^{BE} = I_B^B \omega^{BB'} + I_B^B \omega^{B'E} \tag{3.9}$$

The term with rotational derivative, the first term in Equation 3.7 in the left hand side, takes the following form

$$\begin{aligned} D^{B'} l_B^{BE} \quad &= D^{B'}\left(I_B^B \omega^{BB'}\right) + D^{B'}\left(I_B^B \omega^{B'E}\right) \\ &= I_B^B D^{B'} \omega^{BB'} + I_B^B D^{B'} \omega^{B'E} \end{aligned} \tag{3.10}$$

Since rocket is rotationally symmetric, it has been accepted that the expressions $\omega^{BB'} D^{B'} I_B^B$ and $\omega^{B'E} D^{B'} I_B^B$ disappear in the Equation 3.10.

If Equations 3.7 and Equation 3.9 are combined using Equation 3.10, the Equation 3.11 is obtained.

$$I_B^B D^{B'} \omega^{BB'} + I_B^B D^{B'} \omega^{B'E} + \Omega^{B'E} I_B^B \omega^{BB'} + \Omega^{B'E} I_B^B \omega^{B'E} = m_B \tag{3.11}$$

The first expression in the left hand side models the change of spin rate and is assumed to be zero. The second term models the change of body primes angular velocity. The third expression refers to gyroscopic coupling.

Since the rocket is rotationally symmetric, $I_2 = I_3$ is accepted. In addition, if the moment of inertia, spin rate $\omega^{BB'}$and the body primes angular velocity $\omega^{B'E}$ are written in the B' frame, the following equations are obtained.

$$[I_B^B]^{B'} = \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_2 \end{bmatrix}^{B'} \quad , \left[\omega^{BB'}\right]^{B'} = \begin{bmatrix} \omega \\ 0 \\ 0 \end{bmatrix}^{B'}, \quad \left[\omega^{B'E}\right]^{B'} = \begin{bmatrix} p' \\ q' \\ r' \end{bmatrix}^{B'} \qquad 3.12$$

The matrix form of Equation 3.11 can be obtained as follows.

$$\begin{bmatrix} I_1\left(\frac{\mathrm{d}\omega}{\mathrm{d}t}\right) \\ 0 \\ 0 \end{bmatrix}^{B'} + \begin{bmatrix} I_1\left(\frac{\mathrm{d}p'}{\mathrm{d}t}\right) \\ I_2\left(\frac{\mathrm{d}q'}{\mathrm{d}t}\right) \\ I_2\left(\frac{\mathrm{d}r'}{\mathrm{d}t}\right) \end{bmatrix}^{B'} + \begin{bmatrix} 0 & -r' & q' \\ r' & 0 & -p' \\ -q' & p' & 0 \end{bmatrix}^{B'} \begin{bmatrix} I_1 & 0 & 0 \\ 0 & I_2 & 0 \\ 0 & 0 & I_2 \end{bmatrix}^{B'} \times \left(\begin{bmatrix} \omega \\ 0 \\ 0 \end{bmatrix}^{B'} + \begin{bmatrix} p' \\ q' \\ r' \end{bmatrix}^{B'} \right)$$

$$= \begin{bmatrix} m_{B1} \\ m_{B2} \\ m_{B3} \end{bmatrix}^{B'} \qquad 3.13$$

The scalar form of rotational equation of motion is obtained by assuming $\left(\frac{\mathrm{d}p'}{\mathrm{d}t}\right) = p' = 0$ since there is no rolling motion in Non-Rolling reference frame.

$$\frac{\mathrm{d}\omega}{\mathrm{d}t} = I_1^{-1} m_{B_1}$$

$$\frac{\mathrm{d}q'}{\mathrm{d}t} = I_2^{-1}\left(-I_1 \omega r' + m_{B_2}\right)$$

$$\frac{\mathrm{d}r'}{\mathrm{d}t} = I_2^{-1}\left(I_1 \omega q' + m_{B_3}\right) \qquad 3.14$$

Thus, the vehicle's roll degree of freedom does not couple with pitch and yaw rates. Yaw rate r' affects the pitch degree of freedom and pitch rate q' affects yaw degree of freedom.

In addition, Equation 2.13, which is found as the attitude equation, will be as follows, since $p' = 0$,

$$\begin{pmatrix} \dot{q}_0 \\ \dot{q}_1 \\ \dot{q}_2 \\ \dot{q}_3 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 0 & 0 & -q' & -r' \\ 0 & 0 & r' & -q' \\ q' & -r' & 0 & 0 \\ r' & q' & 0 & 0 \end{pmatrix} \begin{pmatrix} q_0 \\ q_1 \\ q_2 \\ q_3 \end{pmatrix} \qquad 3.15$$

The outputs of the rocket modeled in the Non-Rolling Frame can be seen for the same flight conditions.

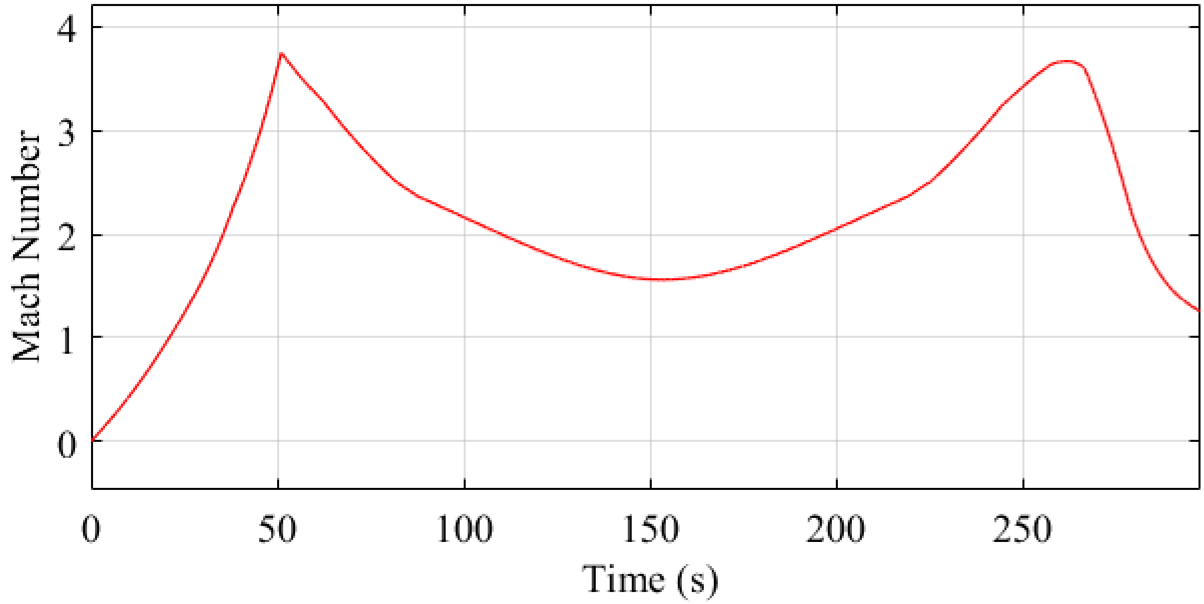

Figure 3.2.10: Mach number over time for 1 Hz spin rate Non-Rolling Body frame model.

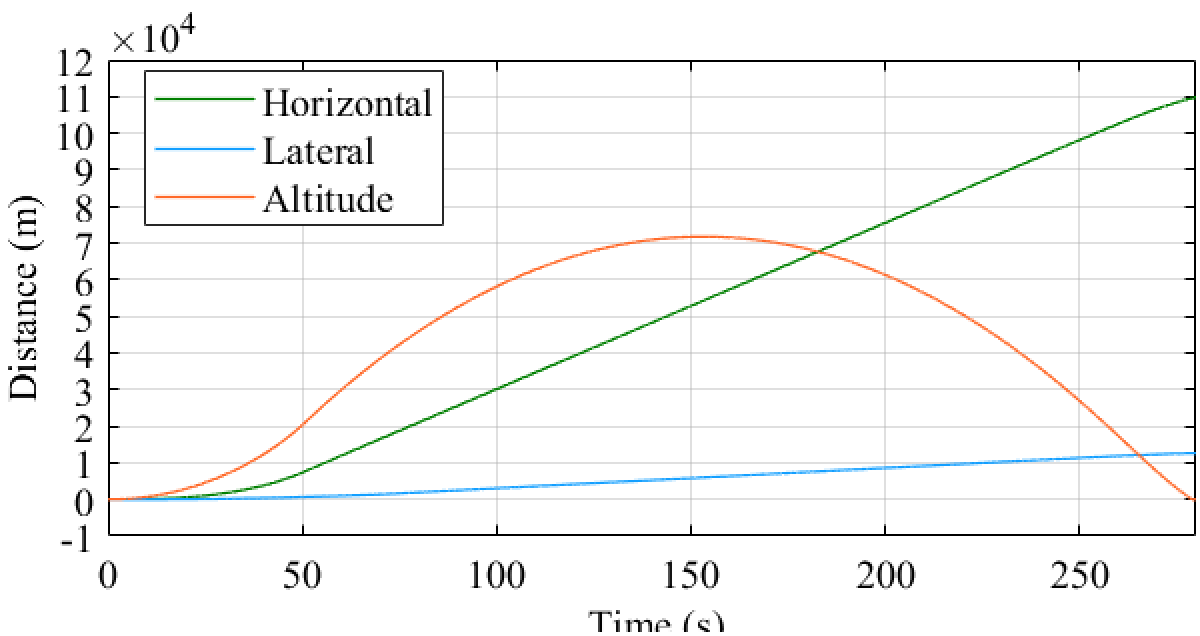

Figure 3.2.11: Distance travelled in each axes for 1 Hz spin rate Non-Rolling Body frame model.

The Mach number shown in Figure 3.2.10 is similar to the results with the of the dynamic equations written in the Body frame seen in Figure 3.2.8. The maximum altitude is decreasing according the results shown in Figure 3.2.11. The distance covered horizontally goes a further. As a result, the two models are not exactly the same.

Figure 3.2.12 and Figure 3.2.13 shows the altitude and speed graphs of the Aerobee 150A rocket in the literature are as follows.[10]

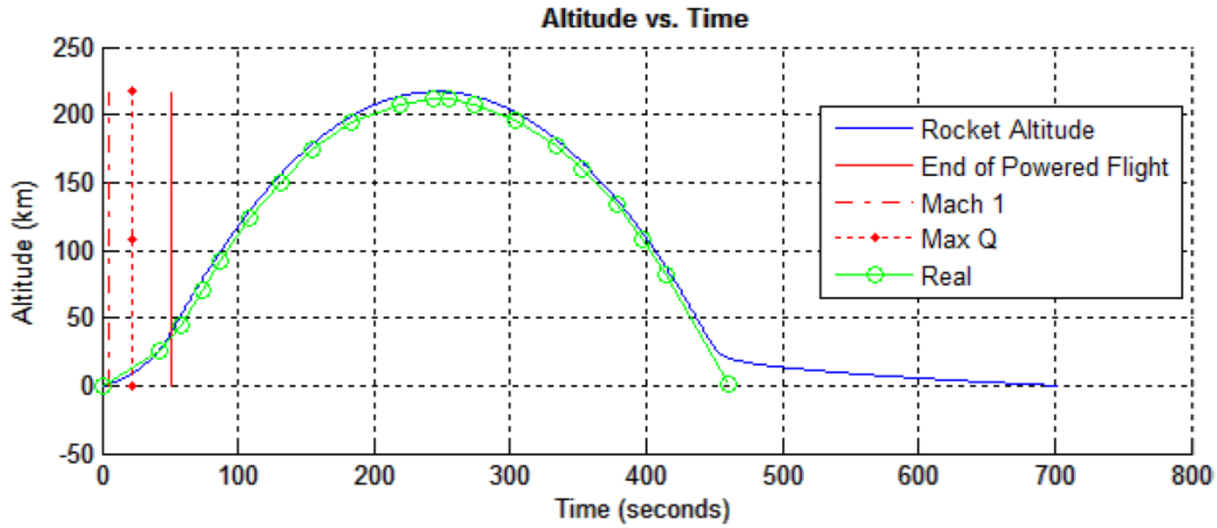

Figure 3.2.12: Altitude over time.

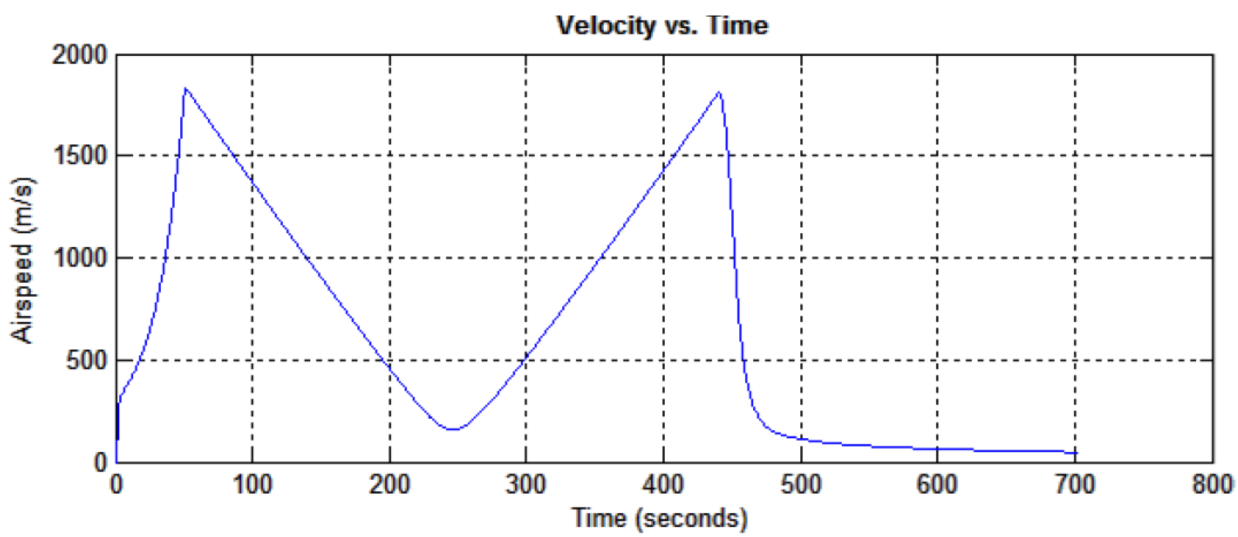

Figure 3.2.13: Airspeed over time.

The highest probability for the reason why the maximum altitude is less than the results in the literature is that the first neglected stage of the rocket. The aerodynamic database created may be missing or incorrect. In addition, the errors that may occur may have caused these differences, since many steps were changed during the development of the model.

4.CONCLUSION AND FUTURE WORKS

Rocket dynamic equations were obtained in Body and Non Rolling Body Frames. Aerodynamic coefficients required for dynamic equations were obtained thanks to DATCOM software. Environmental models and propulsion were modeled. The resulting 6DOF model was simulated on the Simulink platform. It was observed that the output of a rocket with spin rate was better than a non-spinning rocket. It was observed that the rocket dynamic equations written in the Non-Rolling Body Frame were more suitable for the rolling airframe rocket than the Body frame. The outputs were compared with the Aerobee 150A. The reasons for the differences were examined.

The design of a controller based on the 6DOF rocket model obtained within the scope of this thesis can be given as an example of future works. The rocket model obtained can be linearized and linear controllers can be designed. The gain scheduling method with different controllers according to different trim conditions can be an example. If the controller to be designed will be nonlinear, the controller can be designed with the nonlinear robust control method. Thus, the negative effects of the uncertainties in the model can be tolerated to a certain level.

Rolling airframe rocket used in the scope of the thesis can be controlled by controlling yaw and pitch movements only thanks to the Non-Rolling Body frame in which dynamic equations are written. For this reason, the rocket can be controlled like an aircraft that makes skid to turn maneuver. In this way, the controller will be kept simple.

5.REFERENCES

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6.APPENDICES

Missile DATCOM Input File

```
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DIM M
DERIV RAD
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                  1.1,1.2,1.5,2.,3.,4.,5.,6.,7.,8.,9.,10.,
        ALT=7.,70.,200.,500.,630.,700.,790.,880.,1.E03,1.2E03,2.E03,3.7E03,
           7.5E03,1.11E04,1.55E04,2E04,2.53E04,3.1E04,3.88E04,4.8E04,
        NALPHA=20.,ALPHA=0.,1.,3.,5.,8.,11.,13.,15.,17.,19.,
                  21.,25.,30.,35.,40.,50.,60.,70.,80.,89.,
        PHI=0.,$
 $REFQ SREF=0.1140,LREF=0.381,$
 $AXIBOD LNOSE=2.230,DNOSE=0.381,LCENTR=5.11,$
 $FINSET1 XLE(1)=2.63,XLE(2)=2.93,NPANEL=4.,PHIF=45.,135.,225.,315.,
          SWEEP=0.,STA=1.,
          CHORD(1)=0.3,CHORD(2)=0.,SSPAN(1)=0.1405,SSPAN(2)=0.5495,$
 $FINSET2 XLE(1)=6.2986,XLE(2)=6.9716,NPANEL=4.,PHIF=45.,135.,225.,315.,
          SWEEP=45.,STA=1.,SSPAN(1)=0.1405,SSPAN(2)=0.8135,
          CHORD(1)=1.0414,CHORD(2)=0.660,$
BUILD
PART
PLOT
DAMP
SPIN
FORMAT(20(2X,F20.5))
WRITE DB12 1. 400.
WRITE SB12 1. 220.
WRITE SFIN1 1. 220.
SAVE
NEXT CASE
```

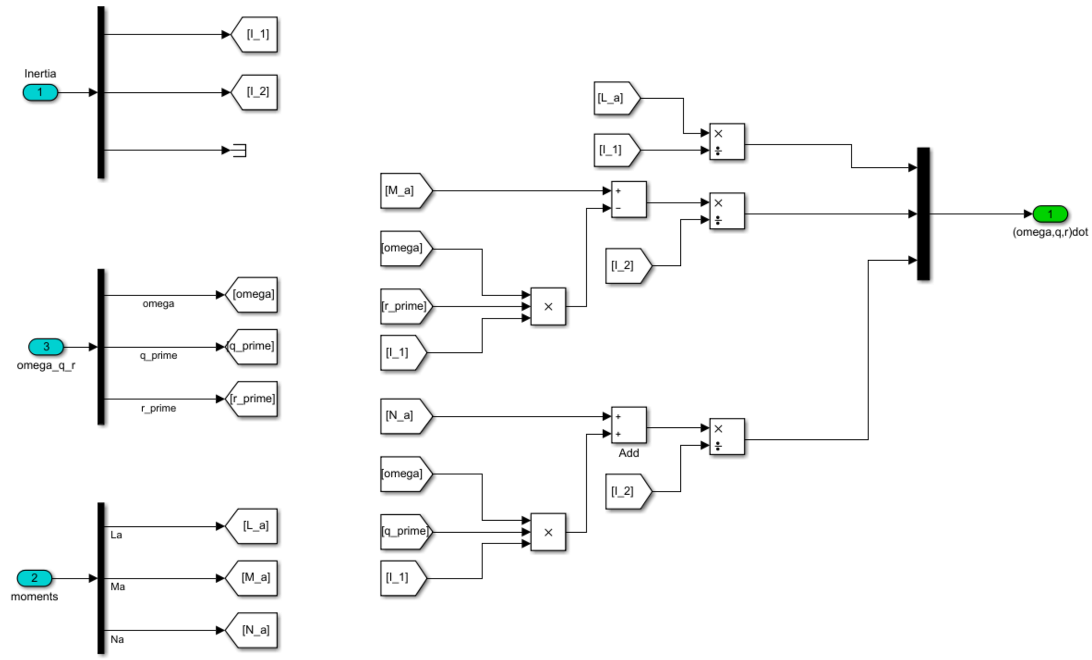

Figure A-1: Rotational Equations Block

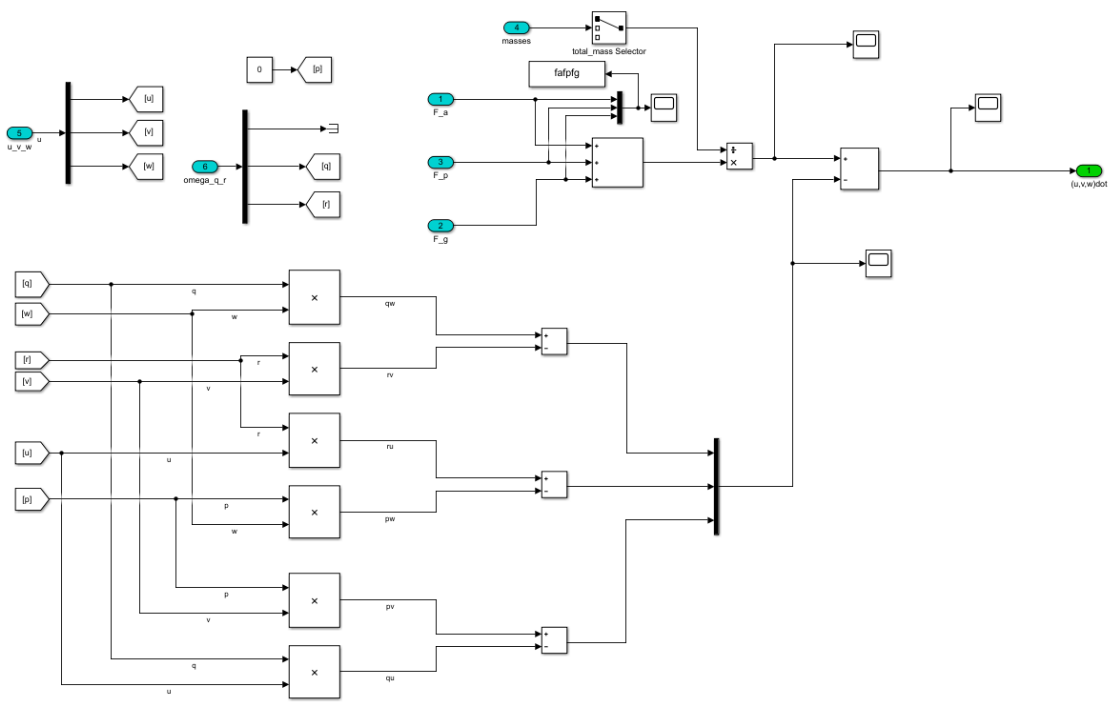

Figure A-2: Translational Equations Block.

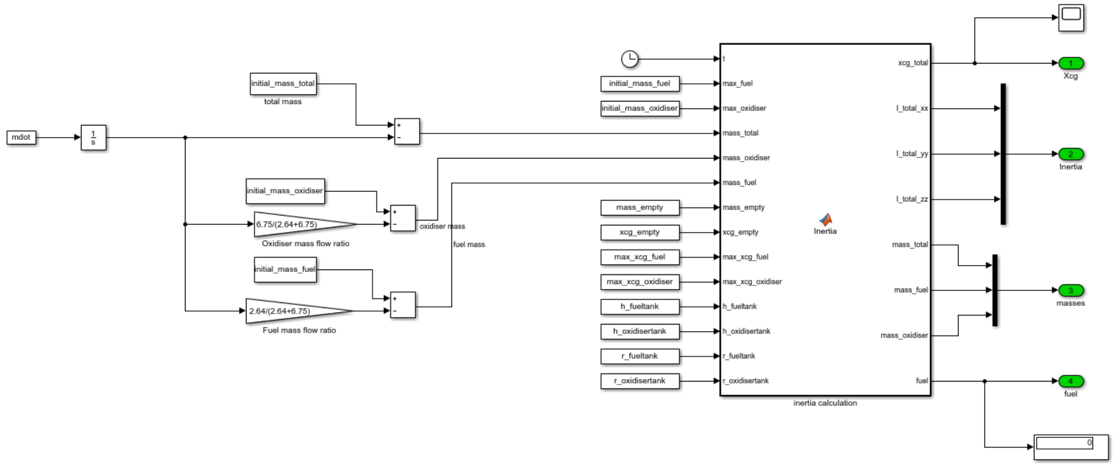

Figure A-3: Mass and Inertia Calculation Block.

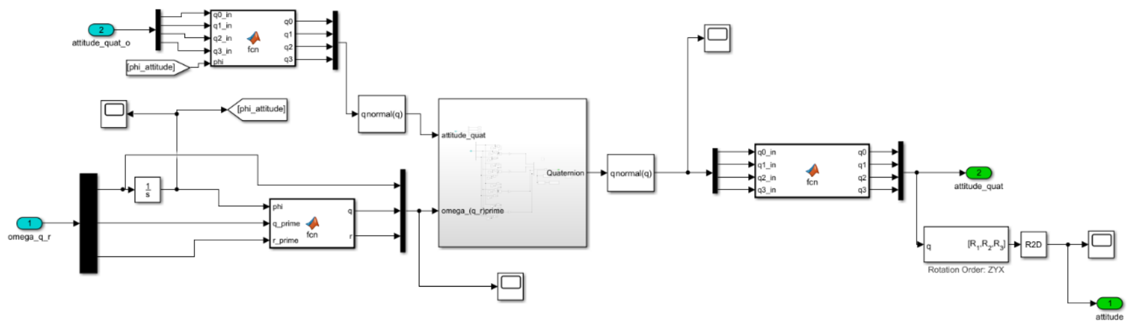

Figure A-4: Attitude Calculation Block

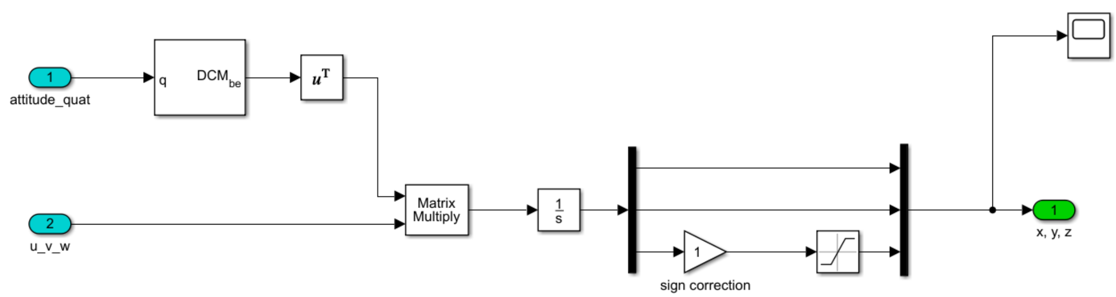

Figure A-5: Displacement Calculation Block.

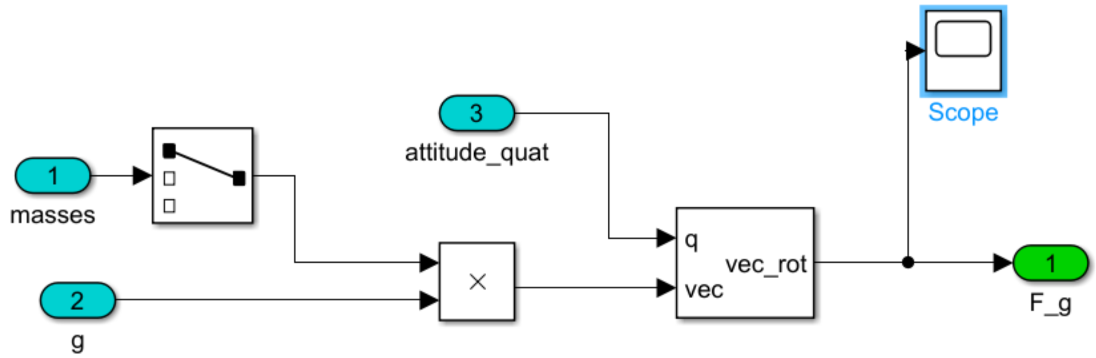

Figure A-6: Gravitational Force Calculation Block.

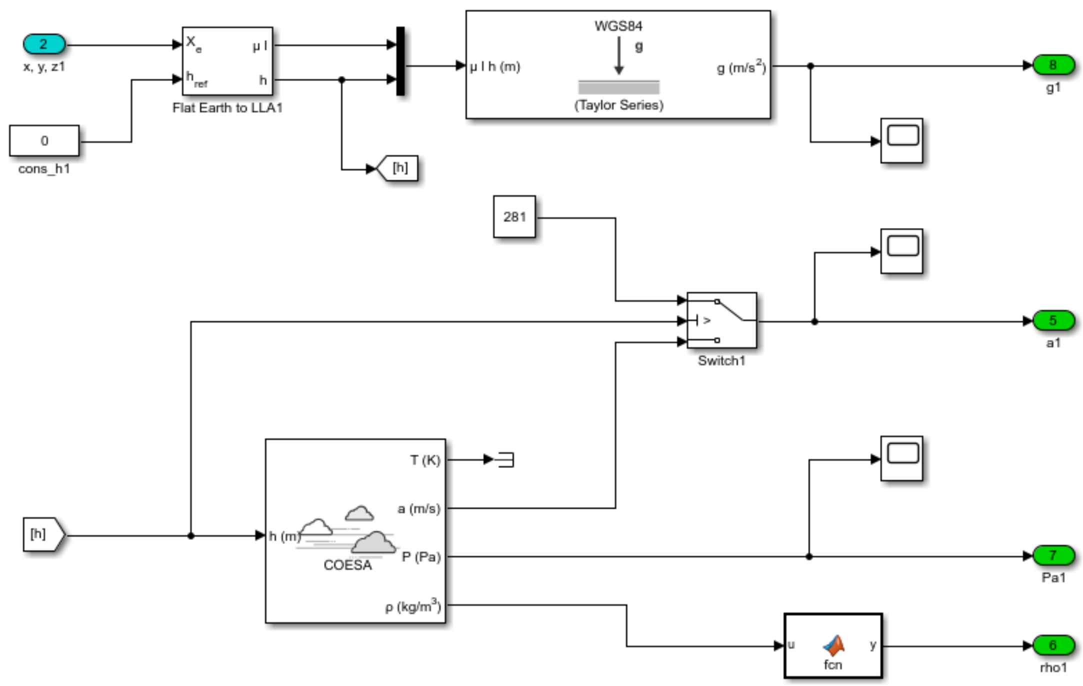

Figure A-7: Environmental Factors Block.

CURRICULUM VITAE

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Date of Birth : 9/24/1996
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Education

University : Aeronautical Engineering, Istanbul Technical University 2015 – Present
High School : Istanbul Ataturk High School of Science 2010-2014

Experience

ITUNOM UAV Team-Subteam Leader (2018 – Oct.2019)
ITUNOM UAV Team-Member (June 2017 – Oct.2018)

Achievements

2019 : 1st place at Teknofest 2019 Fighter UAV Competition
8th place at AUVSI SUAS 2019 Competition
2018 : 7th place at AUVSI SUAS 2018 Competition

Skills

Computer : Matlab and Simulink
CATIA
Solidworks
Ansys Mechanical
MS Office (Word, Excel)