ISTANBUL TECHNICAL UNIVERSITY ★ FACULTY OF AERONAUTICS AND ASTRONAUTICS

FORMING A MATHEMATICAL MODEL FOR AIRCRAFT GROUND DEICING USING JET FLOW

GRADUATION PROJECT

Yiğitcan AYVAZ

Department of Aeronautical Engineering

JULY, 2020

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Yiğitcan Ayvaz,student of ITU Faculty of Aeronautics and Astronautics student ID **110150030**, successfully defended the **graduation** entitled “**FORMING A MATHEMATICAL MODEL FOR AIRCRAFT GROUND DEICING MODEL USING JET FLOW”** which he prepared after fulfilling the requirements specified in the associated legislations, before the jury whose signatures are below.

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Date of Submission : 13 July 2020
Date of Defense : 20 July 2020

To my family,

FOREWORD

This thesis is the final work of my Bachelor study at the Istanbul Technical University. It serves as documentation of my research during the study, which has been made from February 2020 until July 2020. It presents the results of a study towards aspects of aircraft ground deicing methods. It specifically looks at deicing using with preheated fluids.

It was a true learning experience, I had to scan many sources through several fields; but it was also a road paved with delays and unexpected problems.

For his help, I would like to thank my thesis advisor: A. Cihat Baytaş, for his advices and feedbacks, also for his efforts to communicate with me. I also would like to thank my parents for providing me a comfortable environment to study effectively.

July 2020 Yiğitcan AYVAZ

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ABBREVIATIONS

$\boldsymbol{T_a}$: Ambient temperature
$\boldsymbol{A}$: Area of the surface
$\boldsymbol{h_4}$: Coefficient of energy convection between fluid and environment
$\boldsymbol{\rho_2}$: Density of ice
$\boldsymbol{\lambda_f}$: Deicing fluid energy conduction coefficient
$\boldsymbol{\rho_f}$: Fluid density
$\boldsymbol{F_f}$: Fluid flow rate
$\boldsymbol{c_f}$: Fluid specific heat
$\boldsymbol{T_f}$: Fluid temperature
$\boldsymbol{h_5}$: Latent heat of the ice melting
ε : Radiation rate of deicing fluid
$\boldsymbol{c_2}$: Specific heat of ice
σ : Stefan-Boltzmann constant
$\boldsymbol{T_s}$: Surface temperature
δ : Thickness of fluid layer
$\boldsymbol{t}$: Time
$\boldsymbol{V_f}$: Total fluid volume

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FORMING A MATHEMATICAL MODEL FOR AIRCRAFT DEICING USING JET FLOW

SUMMARY

The goal of this graduation thesis *Forming a mathematical model for aircraft ground deicing using jet flow* is to analyze the deicing process and identify its main factors. These factors, namely fluid temperature, fluid flow rate and injection time, are then modified to observe how they change the deicing efficiency.

The graduation thesis is composed of five chapters, each of them dealing with different aspect of deicing process.

Chapter 1 is introductory and describes the aircraft icing phenomenon and how it is taken care of. The chapter is subdivided into two parts. Part 1 describes how the deicing process is initiated and explains the aerodynamic degradation caused by icing. Part 2 provide an overview of the physics of ice accumulation on wings and various ground deicing methods.

Chapter 2 suggests a mathematical model for deicing system. The chapter consists of three parts. Part 1 outlines the deicing using jet flow method. Part 2 investigates the heat transfers in the system to fully capture the nature of deicing. Part 3 defines the time factor and binds all the heat transfers to form a mathematical model.

Chapter 3 is subdivided into three parts and provides an outline of numerical solution for deicing model, since it is not analytically solvable. Part 1 assigns the boundary and environmental conditions in order to realize the mathematical model. Part 2 explains the used numerical method, Runge-Kutta methods and implementations to increase its accuracy. Part 3 informs about how to code Runge-Kutta methods.

Chapter 4 concentrates on the results from various trials. The chapter consists of two parts. Part 1 compares the results and emphasize the roles of different parameters; fluid temperature, flow rate and injection time. Part 2 provides an overview of how these parameters alter the ice melting rate.

The results of the computer trials show that the flow rate and the initial temperature of deicing fluids virtually have the same influence over deicing process, both of which can alter the melting rate and total volume of melt ice by modifying the surface balance temperature. The injection time can vary the surface balance temperature phase duration. Nevertheless, the value of the equilibrium temperature cannot be changed while it can only affect the total melt ice volume and has no effect on the rate.

Conclusions are drawn in Chapter 5. The main aim of the graduation thesis has been reached. On this chapter, the author shares his contribution, and how this study can be improved in the future.

ÖZET

Bu mezuniyet tezinin amacı, yerde duran bir uçakta jet akışı kullanarak buz çözme işlemini temsil eden matematiksel bir model oluşturmaktır. Bunun için buz çözme süreci analiz edilmiştir. Sürecin ana faktörleri; sıvı sıcaklığı, sıvı akış debisi ve püskürtme süresi olarak tanımlanmış olup, ardından bu faktörlerin buz çözme hızını nasıl etkilediği araştırılmıştır.

Bu mezuniyet tezi beş bölümden oluşmuş ve her bir bölümde buz çözme süreci farklı açıdan incelenmiştir.

Birinci bölüm tanıtım niteliğindedir ve uçakta buzlanma olgusunu ve bu durumla nasıl başa çıkıldığını açıklamaktadır. Bölüm iki kısma ayrılmıştır. Birinci kısımda buz çözme işleminin nasıl başlatıldığı ve buzlanmanın neden olduğu aerodinamik performans düşüklüğünü açıklanmıştır. İkinci kısımda kanatlarda buz birikimi sürecini fiziksel olarak açıklanmış ve çeşitli yerde buz çözme yöntemlerine dair genel bir bakış sunulmuştur.

İkinci bölüm buz çözme sistemi için bir matematiksel model önermektedir. Bölüm üç kısımdan oluşmaktadır. Birinci kısımda jet akış yöntemini kullanarak buz çözme ana hatlarıyla açıklanmaktadır. İkicni kısımda buz çözmenin sürecinin doğasını tamamen anlayabilmek için sistemdeki ısı transferlerini belirtilmiştir. Üçüncü kısımda ısı transferleri zamana bağlı olarak tanımlanmış ve matematiksel bir model oluşturmak için birbirlerine bağlanmıştır.

Üçüncü bölüm üç kısma ayrılmıştır ve bu bölümde buz çözme modeli analitik olarak çözülemediği için, kullanılacak sayısal çözümün bir taslağını sunulmaktadır. Birinci kısımda matematiksel modeli gerçekleştirmek için sınır ve çevresel koşullar belirlenmiştir. İkinci kısımda, kullanılan Runge-Kutta sayısal yöntemini ve bu yöntemin doğruluğunu arttırmak için yapılan uygulamaları açıklanmaktadır. Üçüncü kısımda Runge-Kutta yöntemlerini kullanan bir bilgisayar kodunun prensipleri hakkında bilgi verilmiştir.

Dördüncü bölümde çeşitli buz çözme örneklerinden elde edilen sonuçlara odaklanılmıştır. Bölüm iki kısımdan oluşmaktadır. Birinci kısımda elde edilen sonuçlar karşılaştırılmış ve farklı parametrelerin (akışkan sıcaklığı, akış hızı ve püskürtme süresi) rolleri vurgulanmıştır. İkinci kısımda bu parametrelerin buz erime hızını nasıl değiştirdiğine dair genel bir bakış sunulmaktadır.

Bilgisayar denemelerinin sonuçları, buz çözücü sıvıların akış debisinin ve kullanılan sıvının başlangıç sıcaklığının, buz çözme işlemi üzerinde hemen hemen aynı etkiye sahip olduğunu göstermiştir. Bunların her ikisi de, yüzeydeki denge sıcaklığını değiştirerek buzun erime hızını ve eriyik buzun toplam hacmini değiştirebilir. Püskürtme süresi, yüzeydeki denge sıcaklığı evresinin süresini değiştirmiştir ancak denge sıcaklığının değerini değiştirememiştir. Ayrıca, erime hızı üzerinde de bir etkisi olmadığı halde toplam eriyik buz hacmini değiştirirmiştir.

Sonuçlar beşinci bölümde verilmektedir. Mezuniyet tezinin temel amacına ulaşılmıştır. Bu bölümde yazar, yerde duran bir uçağın üzerinde oluşan buzun jet akış ile temizlenmesi için bir matematik model geliştirilmesi konusuna katkısını ve bu çalışmanın gelecekte nasıl geliştirilebileceğini paylaşmaktadır.

1. INTRODUCTION

Aircraft icing occurs when steam or humidity in the air rapidly lose heat and freeze and then pile up in time on aircraft's surface. Deicing fluids are used to clean the accumulated frost and water on aircrafts. This procedure is considered as a ground service in Turkey although various airlines deploy their own crews according to different countries' conditions.

Purpose of deicing is to ensure aircrafts' safety at flight. Because icing at wings increase drag force and weight, decrease lift force, and thus severely degrades aerodynamic performance by disrupting airflow around wings. Ice must be removed from crucial surfaces by proper pre-flight deicing operation. Initiating deicing process is pilot's decision, meaning pilots ought to examine the icing properties around the aircraft.

Although ice formation on airframe surfaces of aircraft exposed to frozen precipitation at airports is commonly witnessed aircraft icing incident, this icing hazard can be easily handled by applying de-/anti-icing fluids to the airframe surfaces prior to takeoff (Liu & Hu, 2018).

Deicing fluids are four types and are mainly comprised of propylene glycol. Deicing fluids, sometimes mixed with water, applied on surfaces with high temperature and pressure to accelerate melting process. Multiple numbers of vehicles may operate on same aircraft depending on the size of aircraft.

1.1 Purpose of Thesis

Aim of this study is to form a mathematical model for deicing process using jet flow.

Deicing process has many expenses on airlines including providing, storing, applying chemicals and time aircraft spend on ground. In reality, most airports typically use as many deicing fluids as possible to melt the ice in minimum time, which causes the deicing fluid wastage and ecological contamination.

Consequently, a mathematical model for aircraft deicing using jet flow should be designed to lay the foundations for following researches, such as deicing fluid consumption optimization (Chen et al., 2016)

Firstly, properties of frost and water layer on surface and properties of jet flow are investigated. Then, governing equations of mass and energy are set. Afterwards, same process is repeated using different parameters. Comparing the results, most efficient conditions for deicing process are determined.

1.2 Literature Review

The airline industry significantly contributes to economic growth. It offers rapid and simple transportation of passengers and cargo. Consequently, flights are always in demand no matter the air conditions. This situation encourages researchers to study the frost formations and deicing mechanism on surfaces of aircrafts.

Cao et al. (2018) explain that main cause of aircrafts icing is supercooled water droplets. Snow falling from the sky and liquefying through warm air may lead to supercooled water droplets. This happens in a grander probability when cold and warm air masses collide and form a contrary layer. Supercooled water droplets are volatile, meaning that they will freeze on impact on surfaces. Supercooled water droplets have greater mass and inertia, thus a portion of them will follow the airflow and bump to airplane surface. As a result, supercooled water droplet freeze and form an ice layer, as shown in Fig 1.1.

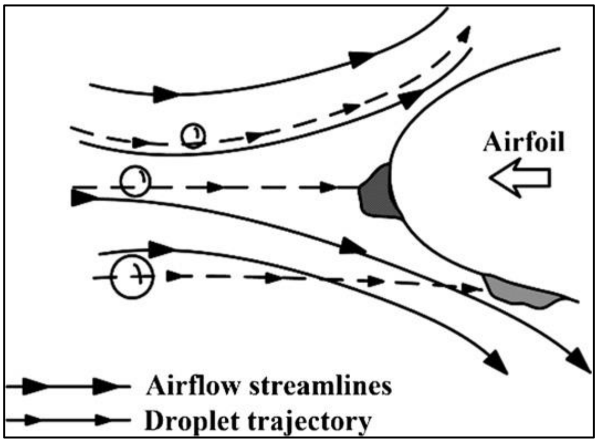

Figure 1.1 : Typical air and water droplet flow around an airfoil. (Cao et al., 2018)

Gent et al. (2000) report surface shape and roughness; total area, airflow speed, temperature, liquid water content and droplet size determine the ice accretion rate and amount. Impacting water droplets must dissipate their latent heat of fusion through convection and evaporation to freeze. Researchers classify aircraft icing into three categories: rime ice, glaze ice and mixed ice (Cao et al., 2018). Rime ice generally show up in a situation of low velocity, low temperature (less than -10 C°), low fluid water content and little droplet diameter, which means water drops can without much of a stretch harden. When the supercooled water drops hit the wing surface, they promptly freeze and stay a hemispherical shape, along these lines framing a spear like ice shape on the leading edge as shown in Fig 1.2

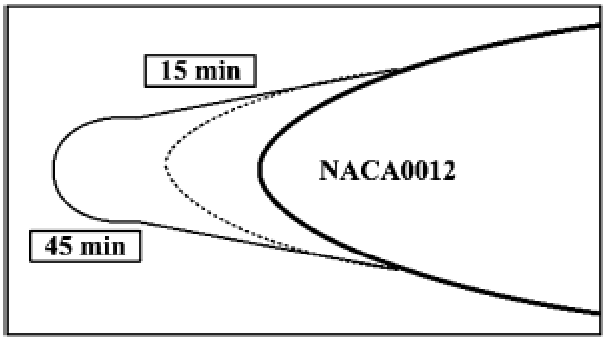

Figure 1.2 : Typical rime ice shape. (Wright, 2002)

In relatively warm conditions (0 to -10 C°) glaze ice is formed. Therefore, it is difficult for water droplets to freeze on the wing surface instantly. Consequently, the droplets flow along the surface until they are either frozen or blown away by aerodynamic forces. Mostly, its shape is a single or double horn as shown in Fig 1.3 because the water overflows along the direction of airflow. Glaze ice predominantly has a greater density and smoother form because water droplets are closely combined. Additionally, glaze ice is harder to remove since it is firmly attached to the aircraft surface.

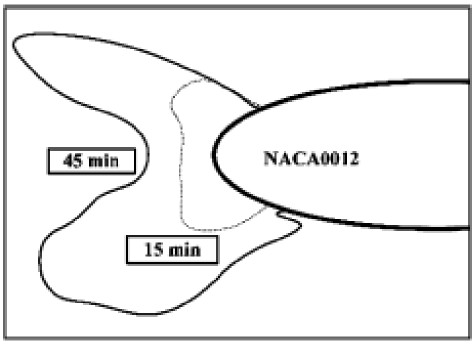

Figure 1.3 : Typical glaze ice shape. (Wright, 2002)

In reality, airplane icing frequently has the trait of a blend of glaze and rime ice with no predetermined ice structure as seen on Fig 1.4 because the atmosphere varies widely in liquid water content and water droplet diameter. Different types of ice can affect flight performance variously and have different material properties.

Figure 1.4 : Typical mixed ice shape. (Icing Hazards, n.d.)

Airplane icing is a significant external reason as indicated by investigations of the causes for the airplane accidents. The bulk of the accidents were fatal. Studying the effects of icing is important given the severe harms caused by aerodynamic degradation. Cao et al. (2018) point out the dangers of aircraft icing. Ice accretions on wings disrupt airflow around wings, which reduces maximum lift and stall angle of attack, while increasing stall angle of attack. At this point, autopilot decrease airspeed to decrease the drag force that engine has to overcome. At this stage, if the pilot fails to pay attention to airspeed and climb rate changes, the aircraft may be near stall boundary.

Gent et al. (2000) clarify that ice can be especially hazardous on wings and tail surfaces during take-off and can cause serious stability and control problems. Ice on airplane sensors can prompt inaccurate information, for example, wrong airspeed indication or mistaken information gave to motor control programming, bringing about inaccurate motor force settings and potentially airplane loss.

Moreover, Chen et al. (2016) emphasize airplane ground ice is the immediate reason for the winter flight postponement, cancelation, and closure of the airport, which can cause tremendous financial damages and furthermore heavily affect general transportation.

Since the very early years of flight, protecting the aircraft from the undesirable effects of ice accretion has been a vital design principle. It was not until the advent of the computer age in the late 1970s that significant progress in the theoretical studies of aircraft ice accretion was made. Gent et al. (2000) declare that foundations of aircraft icing using numerical analysis are provided by the early works of Hardy (1946), Langmuir & Blodgett (1946) and Messinger (1953) which are early primary milestones [5]. In order to simulate the icing process, several ice prediction codes such as LEWICE, IGLOO2D by ONERA, FENSAP and ICECREMO have been developed and compared with experimental results. Liu et al. (2019) informs that simulation of ice accretion is delicate because treating heat conduction in the ice layer as a steady process is not accurate. A schematic of energy balance of ice stack is shown in Fig 1.5 (Liu & Hu, 2018).

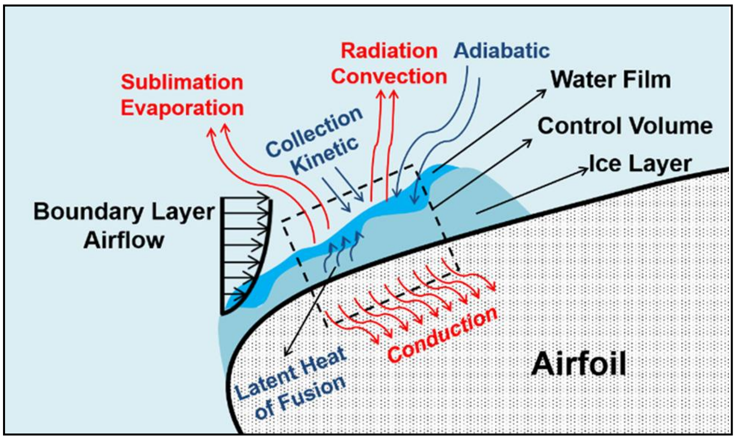

Figure 1.5 : A schematic of energy balance over an ice accreting airfoil surface

Many modern and practical methods of deicing aircraft are arising, such as microwave, infrared and hot air deicing. Yet, aircraft deicing fluids are primarily practiced when it comes to aircraft ground deicing or anti-icing (Chen et al., 2016) Ryerson et al. (2003) state that glycol-fluid deicing technologies are the cornerstones of commercial aviation because they are fast, efficient in most climates therefore enable airlines to maintain schedules. However, the cost of deicing fluids in financial and environmental terms is quite high. Environmental Protection Agency (EPA, 2000) highlights that fluid recovery costs are about \$11–\$13/gallon for the approximately 11 million or more gallons of deicing fluids used annually in the US. Approximately 70 % of the cost per gallon is for recovery, recycling and disposal of used fluid to avoid its leakage into groundwater and surface water and wastewater treatment facilities.

Glycol based deicing fluid is applied entrained in air jet flow. The key point of this type of ground deicing is that jet flow must impinge at an angle and be far enough away from the wing to prevent surface damage (Yakhya et al., 2019). The efficiency with which frozen pollutants are removed from the surfaces of the aircraft is most effectively achieved with a nozzle spray angle of about 45 degrees (Zhu et al. 2015). Hrycak et al. (1970) describe the vertical impingement of the jet flow field in three regions, as shown in Fig 1.6. Region I covers the flow from the nozzle exit to the crest of the potential core. Region II covers the jet path of the flow establishment beyond the core. Region III describes the spreading of the jet in the transverse direction.

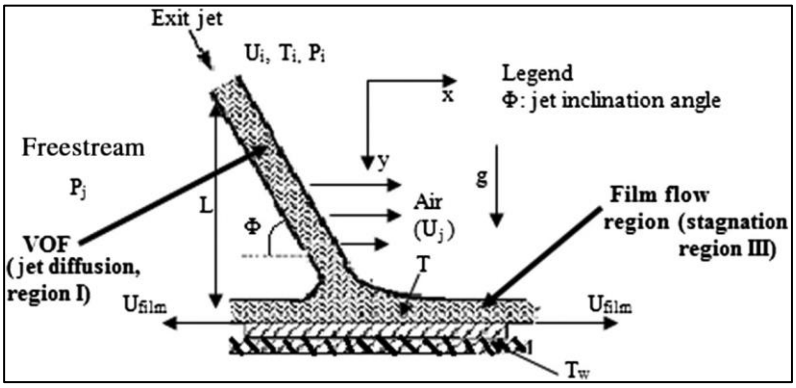

Figure 1.6 : Basic model of liquid jet hitting a flat surface with an angle. (Hrycak et al., 1970)

Chen et al. (2016) explained the situation. As a result, a liquid layer that is slow moving and high-temperature is created. A liquid layer that is slow-moving and high-temperature is created by deicing fluids' rapid sprawl over the surface of the aircraft Based on the heat borne by the liquid film, the ice, snow and frost melt.
Chen et al. set the parameters as flow rate, the injection time of the deicing fluids and the initial temperature. Both the flow rate and the initial temperature can alter the equilibrium temperature, which change the rate and the volume of the melting ice whereas the injection time can only alter the equilibrium temperature period and only affect the volume of the ice melt.

1.3 Hypothesis

In this work, the focal point is ice-melting process, which is definitely a physical reaction. Thus, aforementioned deicing fluids' chemical properties are trivial. Heat is added to the system with preheated deicing fluid and then it is transferred to ice layer. Even though the initial temperatures of fluid and ice are known, amount of heat transferred cannot be calculated directly because it depends on temperature, which depends on transferred heat. This recurrence relation is solved by iterating heat transfer equations using a computer code.

2. MATHEMATICAL MODEL

2.1 Fundamentals of Deicing Process

Aircraft surfaces with ice accretion are showered with preheated deicing fluids. Deicing fluids disperse rapidly across the surface and create a slow-moving and high-temperature liquid layer. Liquid layer, pass heat to ice layer and make it melt. High viscosity and low freezing point of deicing fluid decreases the freezing point of ice-liquid combination for a period. In the meantime, the melting ice and excessive deicing fluid pour down to the ground.

Aircraft ground deicing process is pictured in Fig 2.1.

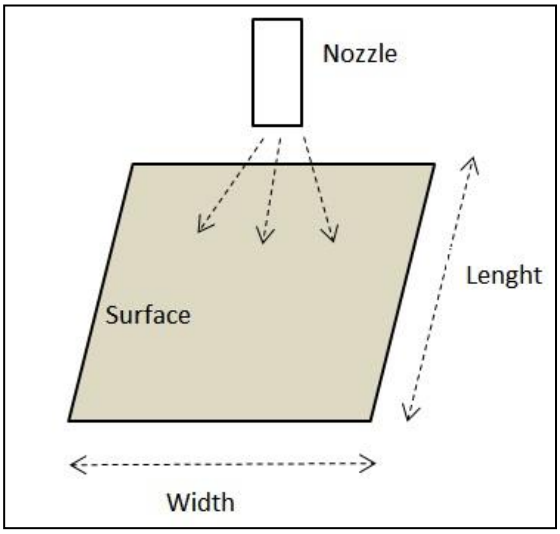

Figure 2.1 : Simplified deicing process.

The airfoil ground deicing process is a classic liquid–solid conversion process, which can be explained using heat transfer and conservation equations. The deicing fluids, ice, and aircraft wing make up a thermodynamic system. The only way to adjust the internal energy of system is through the heat transfer. The sum of heat absorbed from or released to outside is equal to the change of system internal energy (Chen et al., 2016).

Vaporization heat of deicing fluid is ignored, meaning that total volume of spent fluid is entirely liquid. In addition, wind speed is ignored, which can change the heat transfer between fluid and environment by forced convection. Finally, all other energy transfer between system and environment such as fluid to nozzle or wing surface to ice is disregarded. (Myers, 2012)

2.2 Static Modeling of Deicing Process

After these assumptions, the aircraft deicing process can be described as in Fig. 2.2.

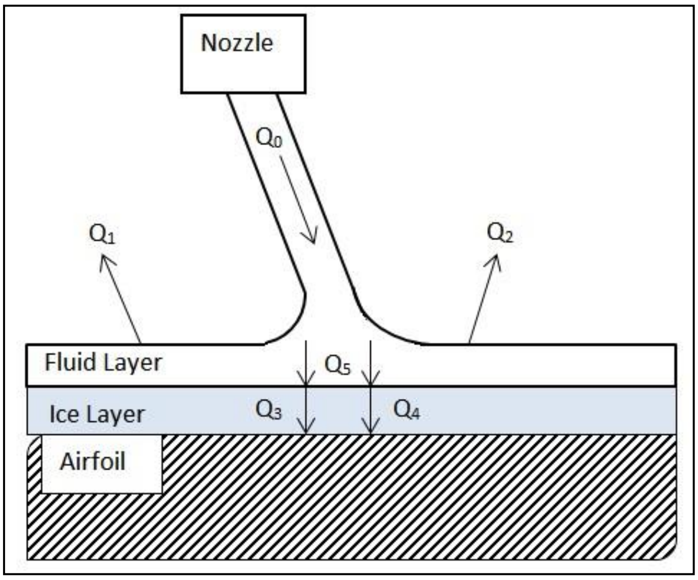

Figure 2.2 : Heat transfers of deicing process.

Q_0 symbolize the energy of deicing fluid applied on the surface.

Q_1 stand for the energy liquid layer lost to environment by convection.

Q_2 indicate the energy liquid layer lost to environment by radiation.

Q_3 represent required to heat the ice from ambient temperature to the melting point.

Q_4 show the energy required to melt the ice on the wing surface.

Q_5 is the transferred between the top and the bottom of the liquid film to ice layer.

The assumptions are as follows:

Fluid layer thickness is considered constant (0.001 m) respect to the viscosity of used fluid. Any excess volume of fluid is assumed to spread around the surface therefore maintaining a constant level, which means volume of deicing fluid is constant in this thermodynamic system.

Deicing fluids' kinetic energy, as a result of speed, is ignored. Hence, the effect of the collision between fluids and ice layer is ignored. Additionally, evaporation of deicing fluids is also ignored despite that they have relatively higher temperature compared to the environment. Thus, any mass (and consequently volume) loss is ignored. Finally, deicing fluids' movement over the surface is considered steady laminar flow.

Fig. 2.3 presents the labeling of ice layer. Arrows show which way distance is measured. Points coincide with surface level, meaning zero ice layer thickness. Also, note that as melting process continues, ice layer surface level decreases thus change in thickness would be negative.

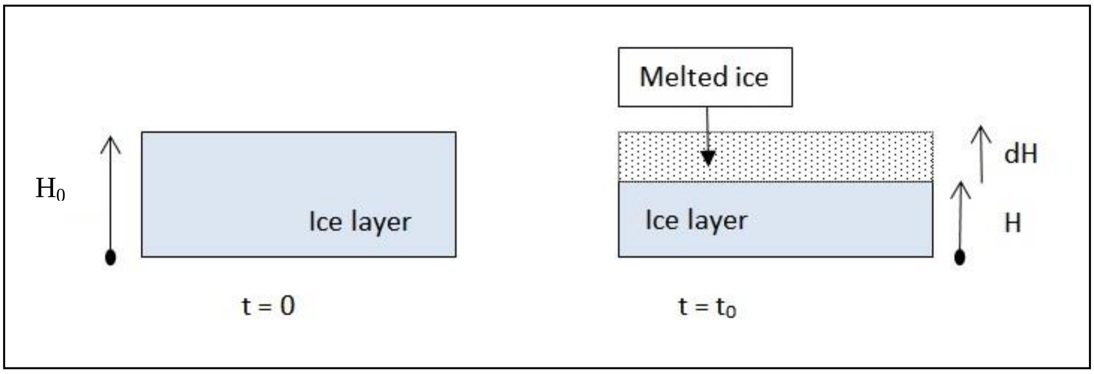

Figure 2.3 : Naming of ice layer.

2.3 Dynamic Modeling of Deicing Process

After going through the assumptions, it can be perceived that only input to this system is heat transferred through deicing fluid because of its elevated temperature. By investigating the heat transfer processes, one can find out the properties of the resulting situation.

2.3.1 Heat Transfer Analysis

Excess heat released form high temperature deicing fluid is only energy input to ice-surface system. The input energy can be stated as Q_0

$$Q_0 = \rho_f c_f V_f (T_f - T_s) = \rho_f c_f F_f t (T_f - T_s) \qquad \mathbf{(1)}$$

where ρ_f is fluid density, c_f is fluid specific heat, V_f is total fluid volume, F_f is fluid flow rate, t is time, T_f is fluid temperature and T_s is surface temperature.
The energy lost to environment with convection from liquid layer is expressed as Q_1

$$Q_1 = \int_0^t Ah_4(T_s - T_a)dt \qquad (\mathbf{2})$$

where A is the area of surface, h_4 (Fitt&Pope, 2001) is coefficient of energy convection, and T_a is ambient temperature. Since deicing fluid temperature is different from surroundings', deicing fluids emit some energy to the environment with radiation.

$$Q_2 = \int_0^t A\varepsilon\sigma(T_s^4 - T_a^4)dt \qquad (\mathbf{3})$$

where ε is the radiation rate of the deicing fluid film and σ is the Stefan-Boltzmann constant, which is equal to $5{,}67 * 10^{-8}\ Wm^2K^4$ (Cengel, 2002).

Melting of ice on the surface can be simplified into two phases. Firstly, temperature of ice is increased to melting point from ambient temperature. When ice layer reach the melting point, second phase starts. On second phase, ice layer start to melt as a result of transferred heat energy.

Energy required to heat the ice layer to melting point is Q_3. If ice layer thickness was constant, than Q_3 would be simply

$$(Q_3)_{simple} = mc\Delta T = (AH\rho_2)c_2(273 - T_a) \quad (\mathbf{4})$$

where H is ice layer thickness, c_2 is the specific heat of ice and ρ_2 is the density of the ice and ice layer initially at ambient temperature. However, ice layer thickness (H) is declining because of ice is melting. Based on Fig. 2.3, H can be stated as

$$H = H_0 - dH \qquad (\mathbf{5})$$

where H_0 is initial ice layer thickness and dH is change in thickness i.e. time derivative of thickness. Substituing equation (5) into equation (4),

$$Q_3 = A\rho_2c_2(273 - T_a)\left(H_0 - \int_{t=0}^{t=t_0} dH\right) \quad (\mathbf{6})$$

Note the integral boundaries at equation (6). Relation between time (t) and ice thickness (H) is not found yet however, we only need to define the change in H.

Equation (6) defines Q_3 related to $H = H_0 - dH$ variable. It is undesired because H_0 is not an essential value; it changes with parameters beyond the scope of this paper. Focus of this study is melted ice thickness dH (i.e. H'). As a result, H_0 is donated zero to emphasize only melted ise thickness. This way, calculated Q_3 is only related to dH.

$$Q_3 = A\rho_2 c_2 (273 - T_a)\left(-\int dH\right) \quad \mathbf{(7)}$$

Minus sign might be confusing, as it implies negative energy. However, it simply refers to the direction of dH. Again, referring to Fig. 2.3, from the arrow's direction it can be said that going upward mean positive and going downwards mean negative. Naturally, melting process starts at ice surface and proceeds to go downwards, meaning negative direction. In the end, Q_3 becomes positive.

After the ice layer reach melting point, required heat to melt the ice is Q_4. Same process with H is applied again.

$$Q_4 = A\rho_2 h_5 \left(-\int dH\right) \qquad \mathbf{(8)}$$

where h_5 is the latent heat of the ice melting and H is thickness of ice layer.

As mentioned before, after starting to spray, deicing fluids form a layer as seen on Fig. 2.2. If this fluid layer is further examined, it can be concluded that the top layer and the bottom layer would have a temperature gradient. To elaborate, the bottom layer of fluid is constantly in contact with ice layer and losing heat while top layer is fed with 'fresh' high temperature fluid. This temperature gradient cause heat conduction between top and bottom of the fluid layer which can be expressed as Q_5

$$Q_5 = \int_0^t \frac{\lambda_f A (T_s - 273)}{\delta} dt \qquad \mathbf{(9)}$$

Where λ_f deicing fluids' energy conduction coefficient and δ is the thickness of fluid layer, which can be assumed constant in a brief time.

Total energy of this ice-fluid system can be stated as in equation (10), according to conservation of energy principle.

$$Q_0 = Q_1 + Q_2 + Q_3 + Q_4 \qquad \mathbf{(10)}$$

Heat balance of ice layer can be formulated as in equation (11)

$$Q_5 = Q_3 + Q_4 \qquad \mathbf{(11)}$$

where Q_5 is the energy transferred from top layer to bottom layer of fluid, then transferred to ice layer. Q_3 is the energy required to bring the ice to melting point and Q_4 is the required energy to melt the ice.

Using the equation (10) and equation (11), equation (12) is found.

$$Q_0 = Q_1 + Q_2 + Q_5 \qquad \mathbf{(12)}$$

2.3.2 Dynamic Modeling

Aircraft deicing have many variables including wind speed, the ambient temperature, the humidity of air and the amount of ice aggregated on the surface (Tao et al., 2012). The problem is that these variables are nonlinear and almost unpredictable. To remove the complexities from the deicing process, only three parameters are used as input and two parameters are used as output.

Input (control) parameters follow as the fluid flow rate F_f, the initial fluid temperature T_f and the injection time t_0 while output (state) parameters are the surface temperature of fluid layer T_s and ice layer thickness H.

Fig. 2.4 show a basis for aircraft deicing dynamic model.

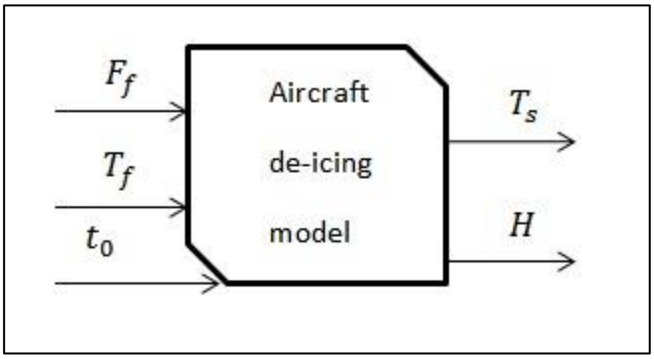

Figure 2.4 : Deicing dynamic model (Chen et al., 2016).

2.3.2.1 Ice Layer Thickness

If equations (7), (8) and (9) are substituted into equation (11), one can find

$$A\rho_2 c_2(273 - T_a)\left(-\int dH\right) + A\rho_2 h_5\left(-\int dH\right) = \int_0^t \frac{\lambda_f A(T_s - 273)}{\delta} dt \qquad \mathbf{(13)}$$

Then, eliminate A and differentiate the obtained equation with respect to time,

$$\left(-\frac{dH}{dt}\right)(\rho_2 c_2(273 - T_a) + \rho_2 h_5) = \frac{\lambda_f(T_s - 273)}{\delta}\left(\frac{dt}{dt}\right) \qquad \mathbf{(14)}$$

one can find the relation between ice thickness H and surface temperature T_s.

$$H' = -\frac{\lambda_f(T_s - 273)/\delta}{\rho_2 c_2(273 - T_a) + \rho_2 h_5} \qquad \mathbf{(15)}$$

H' can be explained as the melting rate. Minus sign means that if surface temperature is higher than freezing point, i.e.$(T_s - 273) > 0$, then ice thickness has a tendency to decrease. Equation (15) states that H' has a positive linear relation with T_s. As a result, if high deicing speed is desired, then the temperature of the surface must be ensured to have a high value.

2.3.2.2 Surface Temperature

After substituting equation (1), (2), (3) and equation (9) into equation (12), this relation is found:

$$\rho_f c_f F_f t(T_f - T_s) = \int_0^t Ah_4(T_s - T_a)dt + \int_0^t A\varepsilon\sigma(T_s^4 - T_a^4)dt + \int_0^t \frac{\lambda_f A(T_s - 273)}{\delta} dt \quad \mathbf{(16)}$$

Differentiate the every element of the equation respect to the time,

$$\frac{d(Q_0)}{dt} = \frac{d(Q_{1)}}{dt} + \frac{d(Q_{2)}}{dt} + \frac{d(Q_{5)}}{dt} \qquad (17)$$

Q_0 have two elements which are related to time, t and T_s. These two must be differentiated separately.

$$Q_0 = \rho_f c_f F_f t(T_f - T_s)$$

$$\dot{Q}_0 = \rho_f c_f F_f \left(T_f - T_s\right) - \rho_f c_f F_f t T_s{}'$$

$$\dot{Q}_1 = \frac{A h_4 (T_s - T_a) dt}{dt} = A h_4 (T_s - T_a)$$

$$\dot{Q}_2 = \frac{A \varepsilon \sigma (T_s^4 - T_a^4) dt}{dt} = A \varepsilon \sigma (T_s^4 - T_a^4)$$

$$\dot{Q}_5 = \frac{\frac{\lambda_f A (T_s - 273)}{\delta} dt}{dt} = \frac{\lambda_f A (T_s - 273)}{\delta}$$

Then combining all,

$$\rho_f c_f F_f \left(T_f - T_s\right) - \rho_f c_f F_f t T_s{}' = A h_4 (T_s - T_a) + A \varepsilon \sigma (T_s^4 - T_a^4) + \frac{\lambda_f A (T_s - 273)}{\delta}$$

$$T_s{}' = \frac{\rho_f c_f F_f \left(T_f - T_s\right) - A h_4 (T_s - T_a) - A \varepsilon \sigma (T_s^4 - T_a^4) - \frac{\lambda_f A (T_s - 273)}{\delta}}{\rho_f c_f F_f t} \qquad (\mathbf{18})$$

Equation (18) shows the dynamical change in the surface temperature of the top fluid layer.

3. NUMERICAL SOLUTION

3.1 Model Interpretation

Dynamical model for deicing process built on two equations: (15) and (18). Major point of equation (18) is the recurrence relation of T_s. On first solution, a $T_s{}'$ value is found, which modify T_s. Then, solving with the new T_s value modifies $T_s{}'$ and cycle continues. Additionally, at every step, melting ice thickness is calculated and accumulated. Modeling this process is achieved using Runge-Kutta methods.

3.1.1 Boundary conditions

At the beginning, ice surface temperature is equal to ambient temperature $(T_s = T_a)$. Because ice is in direct contact with the environment and aircraft is considered to spent enough time on ground so that ice reach the ambient temperature. Then, the moment deicing fluid reach the surface, surface temperature is considered the fluid temperature $\left(T_s = T_f\right)$. After that, heat transfer between fluid and ice start.

Additionally, even though it is physically absurd, ice thickness is taken zero (H=0) to further emphasize the melting ice thickness.

Finally, heat transfer is balanced on the ice surface $(Q_5 = Q_3 + Q_4)$.

3.1.2 Simulation parameters

Parameters used in deicing process may categorize into three: properties of deicing fluids, properties of ice and properties of surface and environment.

Table 3.1 : Properties of deicing fluids.

Parameters	Symbol	Unit	Value
Density	ρ_1	kg/m^3	913
Specific heat	c_2	kJ/kg.K	1.93
Radiation rate	ε		0.28
Thermal conductivity	λ_f	W/m.K	0.3
Film thickness	δ	m	0.001
Convection coefficient	h_4		0.0273 x T_s-3.89

Table 3.2 : Properties of ice layer.

Parameters	Symbol	Unit	Value
Density	ρ_1	kg/m^3	913
Specific heat	c_2	kJ/kg.K	1.93
Latent heat	h_5	kJ/kg	333.5

Table 3.1 : Properties of the surface and environment.

Parameters	Symbol	Unit	Value
Surface area	A	m^2	0.36
Ambient temperature	T_a	°C	-10
Boltzmann constant	σ	Wm^{2}K^4	5.67 x 10^{-8}

3.2 Runge – Kutta Methods

Runge-Kutta methods are a class of iterative methods based on Euler method. They are employed to find approximate solutions of ordinary differential equations with temporal discretization (Devries&Hasbun, 2011). Temporal discretization is a mathematical approach to solve transient problems that require solutions in which a certain parameter varies as a function of time. It involves integrating every term in different equations over a time step. Runge-Kutta methods were developed around 1900 by the German mathematicians Carl Runge and Wilhelm Kutta.

3.2.1 Euler’s Method

Recalling that derivative of a function provides the slope. To find the slope at point x_i, take the first derivative of a function:

$$\emptyset = f(x_i, y_i)$$

where $f(x_i, y_i)$ is the differential equation evaluated at x_i and y_i and $\emptyset$ is the slope. Then using the slope to extrapolate linearly over the step size h a new value of y is predicted:

$$y_{i+1} = y_i + f(x_i, y_i)h$$

This formula is referred to as Euler’s method (Chapra&Canale, 2014) and may be clarified as

$$New\ Value = Old\ Value + Slope * Step\ Size$$

3.2.2. Runge – Kutta methods

Runge-Kutta (RK) methods obtain high accuracy without needing to calculate higher derivatives. RK methods can be generalized as:

$$y_{i+1} = y_i + \phi(x_i, y_i, h)h$$

where $\phi(x_i, y_i, h)$ is called an increment function. It can be read as slope over the interval. The increment is formed as:

$$\phi = a_1k_1 + a_2k_2 + \cdots + a_nk_n$$

where a's represent constants and k's are

$$k_n = f(x_i + p_{n-1}h, y_i + q_{n-1,1}k_1h + q_{n-1,2}k_2h + \cdots + q_{n-1,n-1}k_{n-1}h)$$

where p’s and q’s are constants. Also, note that

$$f(x_i, y_i) = {}^{dy}/_{dx}$$

Notice the recurrence relationship between *k* values: k_1 is used to find k_2, which is used to find k_3 and it continues until k_n. This recurrence relation makes RK methods effective for calculations on computers since each *k* value is a functional estimate and not every term on the right side is needed to be calculated, simply previous *k* value is put to right place.

Runge-Kutta methods are named after the selected *n* value, such as *n*th-order RK. This way, first-order RK method is Euler's method. Depending on *n*, values for constants (a_n, p_n, q_n) are found by setting equal to same n order Taylor series expansion. However, this calculation is beyond the scope of this paper.

3.2.2.1 Runge-Kutta-Fehlberg method

This method is created by German mathematician Erwin Fehlberg and named the RKF45 method (Simos, 1993). It involves using fourth and fifth-order RK with identical k_n values. Then, an estimate of local truncation errors is obtained by subtracting the results from different order RK methods. The error in the solution can be guessed and managed by performing one extra calculation. Then this error value is used for adaptive step sizing.

The advantage of this method is getting smaller error estimates, which means solutions that are more precise are achieved with fewer evaluations. For example, a fourth order prediction requires four evaluations while a fifth order prediction requires five evaluations per step. Then results are subtracted. In the end, this process required a total amount of 10 evaluations. On the other hand, RKF45 method use the same k_n values, meaning only five functional evaluations are needed. Then results are subtracted. Thus, this method yields the error estimate and only required six functional evaluations.

In this paper, the following fourth-order formula is used:

$$y_{i+1} = y_i + (\frac{25}{216}k_1 + \frac{1408}{2565}k_3 + \frac{2197}{4104}k_4 + \frac{-1}{5}k_5)$$

along with the fifth-order formula:

$$y_{i+1} = y_i + (\frac{16}{135}k_1 + \frac{6656}{12825}k_3 + \frac{28561}{56430}k_4 + \frac{-9}{50}k_5 + \frac{2}{55}k_6)$$

where

$$k_1 = f(x_i, y_i)$$

$$k_2 = f(x_i + \frac{1}{4}h, y_i + \frac{1}{4}k_1 h)$$

$$k_3 = f(x_i + \frac{3}{8}h, y_i + \frac{3}{32}k_1 h + \frac{9}{32}k_2 h)$$

$$k_4 = f(x_i + \frac{12}{13}h, y_i + \frac{1932}{2197}k_1 h + \frac{-7200}{2197}k_2 h + \frac{7296}{2197}k_3 h)$$

$$k_5 = f(x_i + h, y_i + \frac{439}{216}k_1 h - 8k_2 h + \frac{3680}{513}k_3 h + \frac{-845}{4104}k_4 h)$$

$$k_6 = f(x_i + \frac{1}{2}h, y_i + \frac{-8}{27}k_1 h + 2k_2 h + \frac{-3544}{2565}k_3 h + \frac{1859}{4104}k_4 h + \frac{-11}{40}k_5 h)$$

To summarize, two solutions are found using fourth and fifth order RK method and error is estimated as the difference between two. In addition, this process is much rather simple because identical k_n values are used.

3.2.2.2 Adaptive step sizing

Runge-Kutta methods use a constant value for step size (h) that is handpicked at the beginning. Deciding the step size is problematic because function nature is yet to be found. To capture the instantaneous changes accurately, smaller step size should be used. However, using smaller step size throughout the whole calculation range mean extra calculations and time. On the other hand, using bigger step size decrease the number of calculations and time needed, but miss the sudden changes.

It would be more convenient to choose the desired accuracy instead of choosing the step size directly (Israel, 2002). Required step size to meet the error criteria can be chosen by the computer code. Error is already estimated on previous topic. Generally, a stepper like this is suggested:

$$h_0 = SF * h_1 * \left(\frac{Desired\ Error}{Calculated\ Error}\right)^{\frac{1}{p}}$$

where h_1 is used step size, h_0 is the appropriate step size, p is the order of solution method and *SF* is the factor of safety, which is usually taken about 0.9.

The 0.9 is a safety factor to ensure success on the next try. Additionally, minimum and maximum values for step size should be also determined to prevent extreme changes.

3.3. Computer algorithm for RK Method

Creating a code to use RK methods require these steps:

1- Select the number of equations, N. In this research, N=1. Formed mathematical model is based on Eq. (15) and (18) but only Eq. (18) has recurrence relation while Eq. (15) solely rely on T_s.

2- Input the initial values for each variable, which are already covered on previous topics.

3- Adjust the code to compute slopes of each variable.

4- Add equations to compute derivative values. These equations are Eq.(15) and Eq.(18)

5- Add loops to compute a new value for each dependent variable.

4. FINDINGS & DISCUSSION

To evaluate the dynamic model successfully, FORTRAN is utilized to solve the non-linear differential equations system using numerical methods. Used parameters are stated on previous chapter, which are coherent with practical use, as Louchez et al. (1998) described.

To start with, a single case of deicing process is examined to emphasize the outlines. The flow rate is $F_0 = 1.0\,L/min$, deicing fluid temperature is $T_f = 60^{\circ}C$ the injection time is 40 s and ambient temperature is $T_a = -10^{\circ}C$. Using these values, simulation is performed and surface temperature and ice thickness curves are shown.

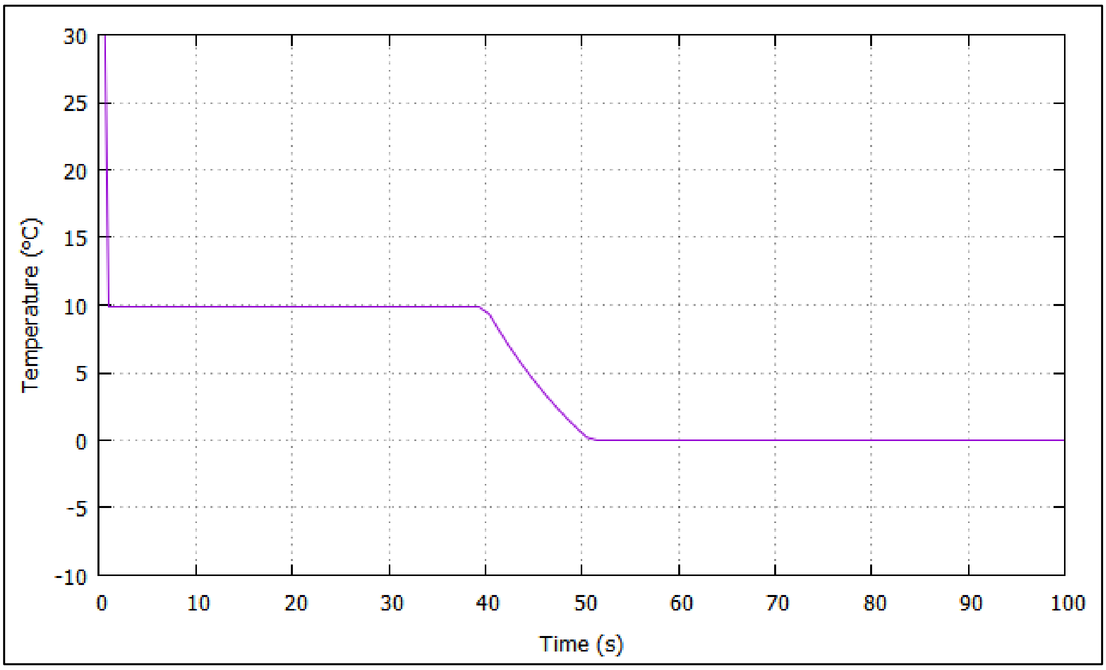

Figure 4.1 : Dynamic simulation of surface temperature change curve.

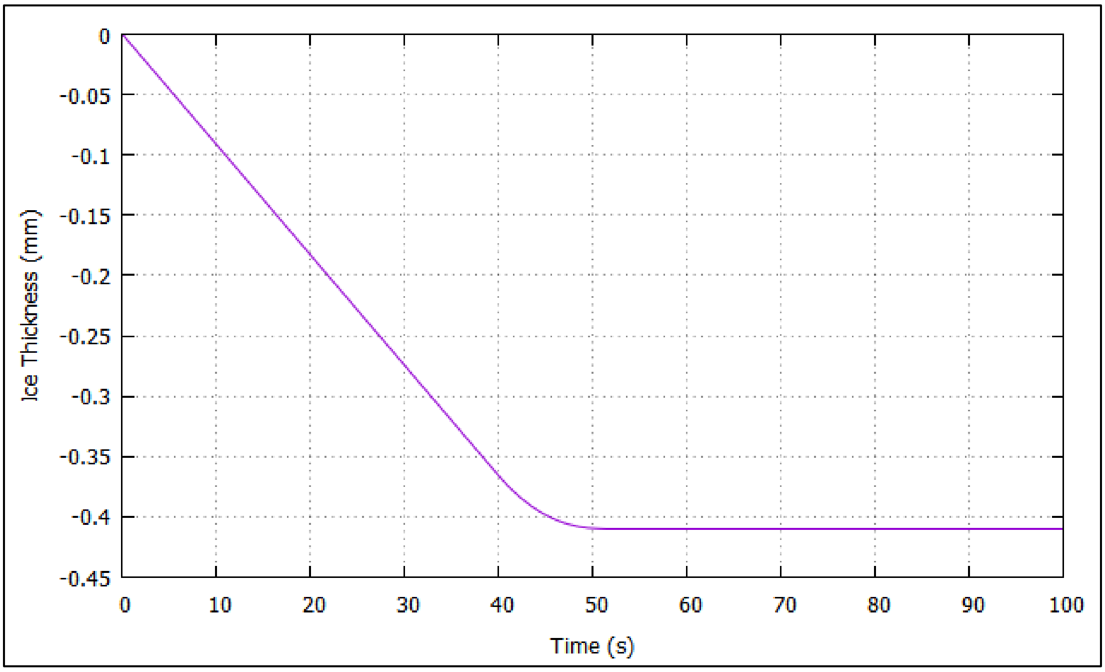

Figure 4.2 : Dynamic simulation of ice thickness change curve.

Surface temperature of deicing fluids, T_s , drops sharply from the initial temperature of $60^{\circ}C$ to the equilibrium temperature $10^{\circ}C$ and stabilize at the equilibrium temperature during the spraying process. After reaching injection time, spraying the deicing fluid stops and T_s start to decrease from equilibrium temperature until the freezing point at 50 seconds.

Ice thickness declines linearly in the balanced temperature stage. After the fluid flow is ceased, the falling rate of ice thickness is lessened, which means ice is melting slowly at this phase. H fixes after 50 s and the total melting thickness is 0.41 mm.

4.1. Significance of Parameters

As mentioned before, the input parameters are the fluid flow rate F_f , the initial temperature T_f , and the injection time t_0 . These parameters are the main elements of deicing process, which are structured by deicing facilities. Throughout the simulation, different variations of these parameters are taken account and their effects are analyzed.

4.1.1. Deicing fluid temperature

The temperature of deicing fluids are introduced as 50°C, 60°C, 70°C, and 80°C. The flow rate is assigned 1.0 L/min and the injection time is set as 40 seconds. Simulating the mathematical model using these values respectively, obtained surface temperature and ice thickness curves are represented below.

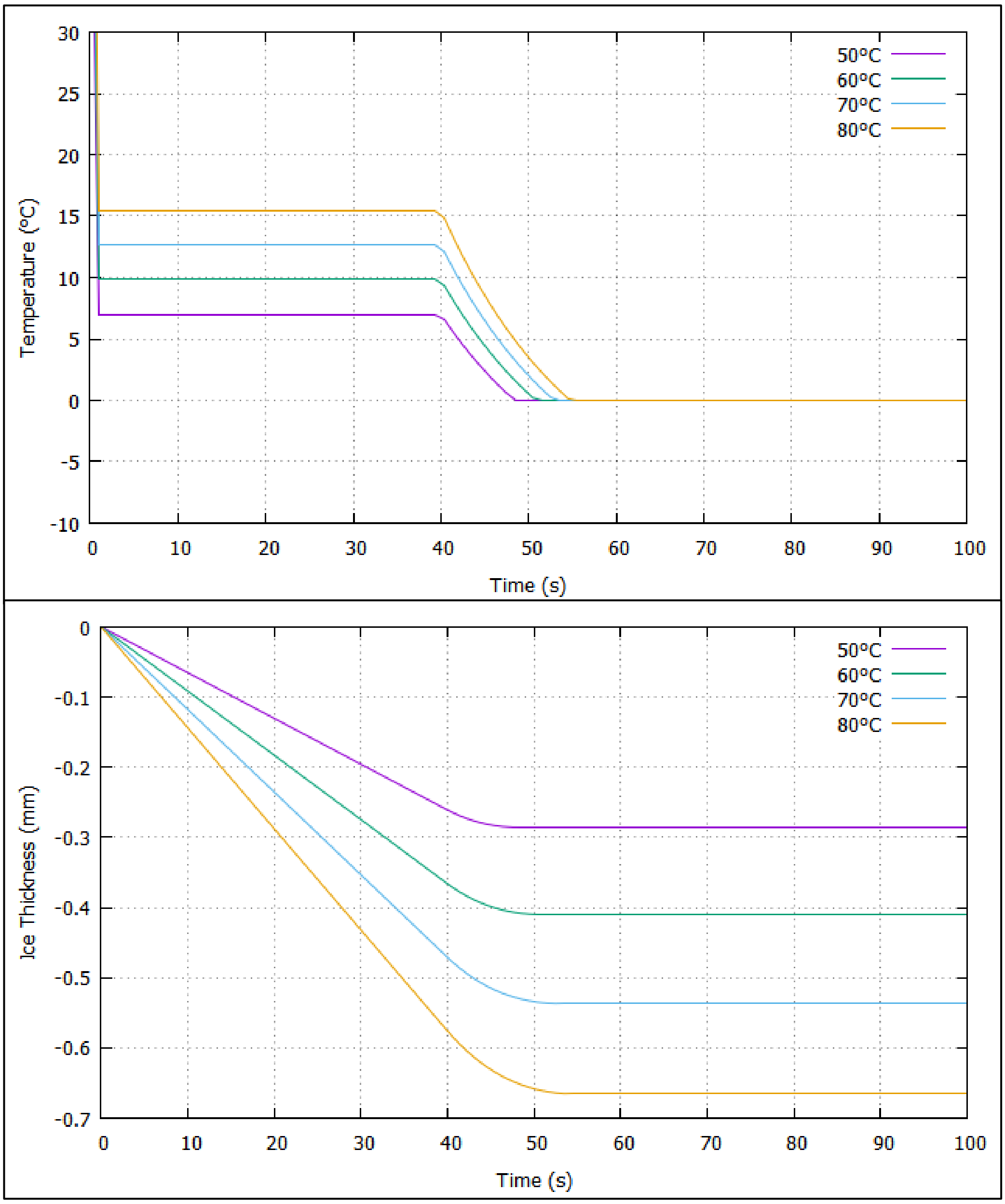

Figure 4.3 : Surface temperature change curves and ice thickness change curves under various initial fluid temperatures.

As seen in Figure 4.3, the balance surface temperature is rising with the increased deicing fluids' initial temperature. Simultaneously, the decrease of ice thickness in the heat balance stage and the overall melting thickness rise with the escalation of the initial temperature. This point out that increasing the initial temperature of the fluids is the key to more potent deicing process.

4.1.2. Fluid flow rate

The flow rate of the deicing fluids is selected as 0.8, 1.0, 1.2, and 1.4 L/min. Other two parameters are selected as: injection time is 40 s and deicing fluid temperature is 60°C. Simulating the mathematical model using these values respectively, obtained surface temperature and ice thickness curves are represented.

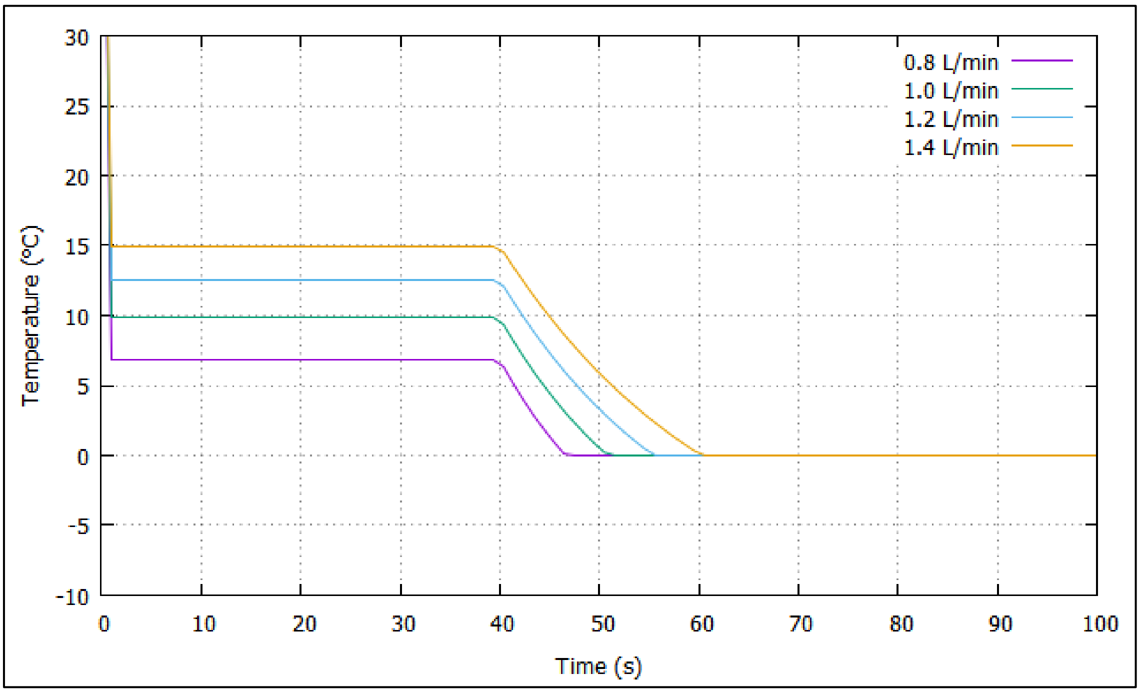

Figure 4.4 : Surface temperature change curves under various flow rates.

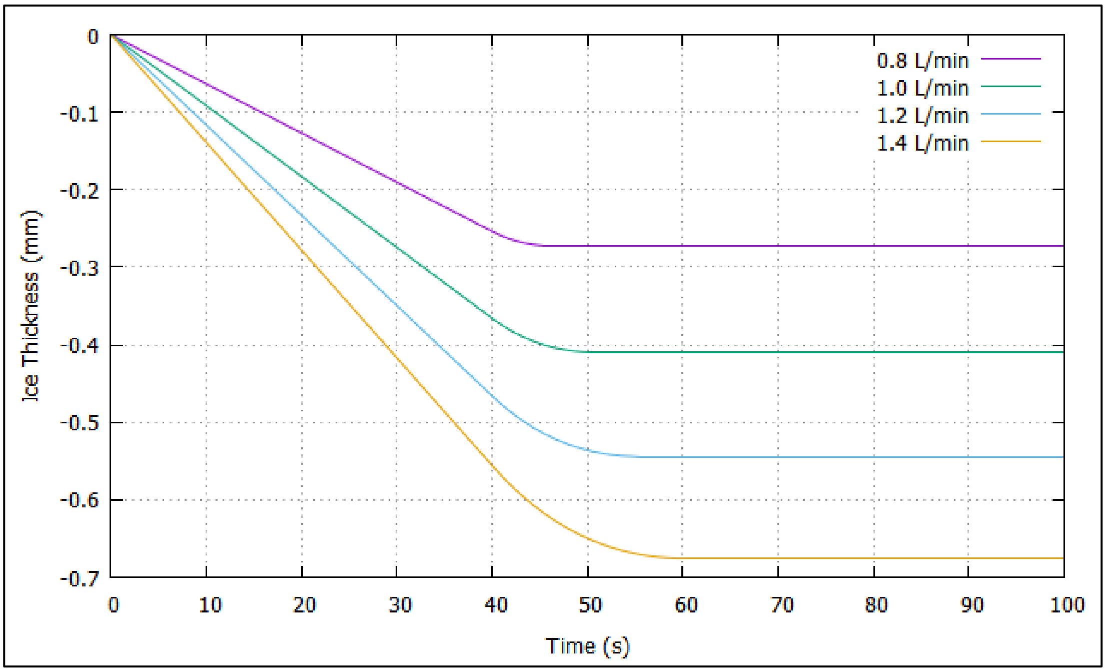

Figure 4.5 : Ice thickness change curves under various flow rates.

As seen in Figure 4.4, bigger the flow rate of the deicing fluids, higher the balance temperature. Concurrently, improving the fluid flow rate also boost the melting speed and overall melt ice thickness. This simulation demonstrates that raising the fluid flow rate can achieve the equivalent control effect as increasing the deicing fluids' initial temperature.

4.1.3. Injection time

The injection time is chosen as 20, 30, 40, and 50 s. The deicing fluid flow rate is 1.0 L / min and the deicing fluid temperature is 60°C. Simulating the mathematical model using these values respectively, obtained surface temperature and ice thickness curves are represented.

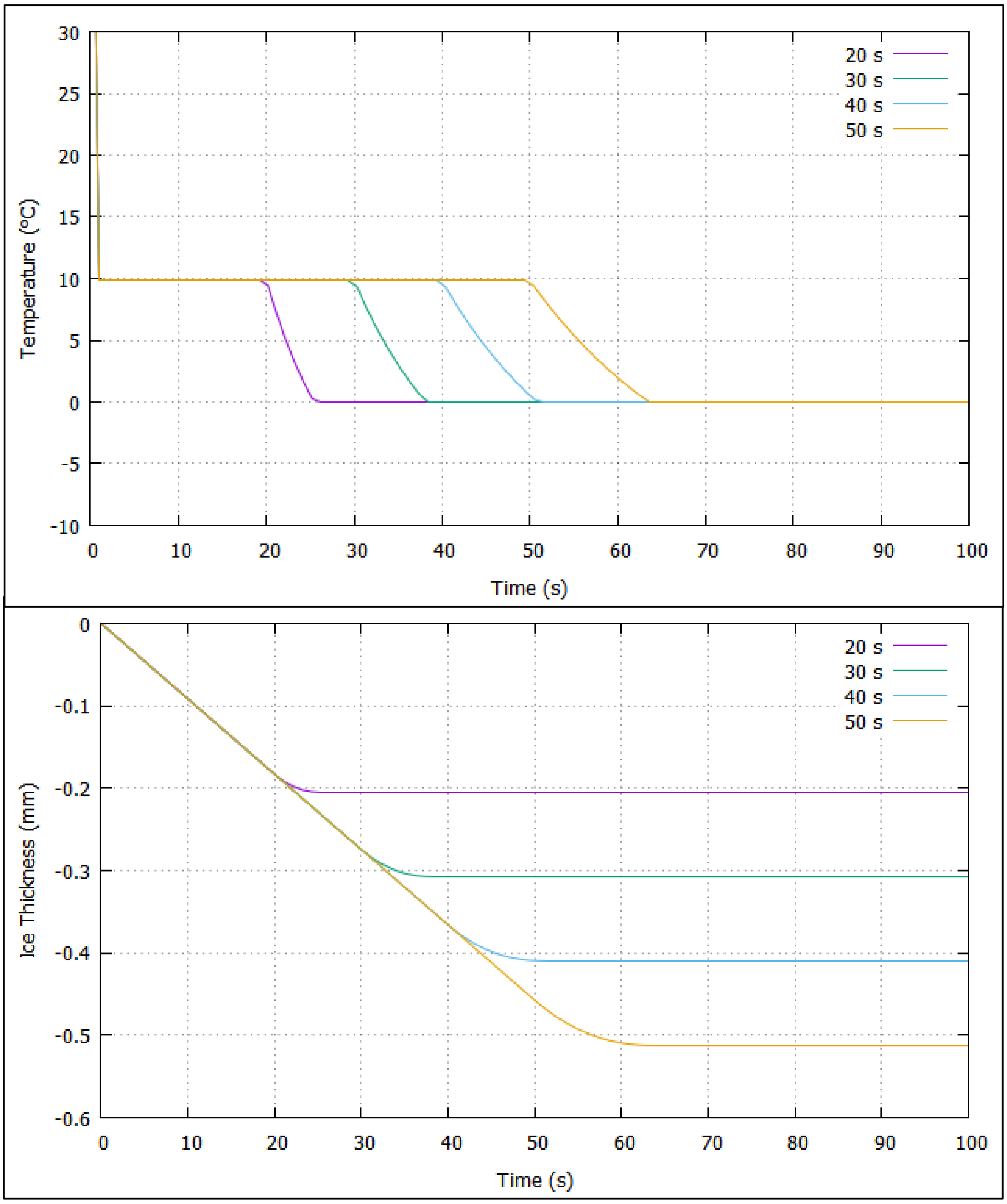

Figure 4.6 : Surface temperature change curves and ice thickness change curves under various injection time conditions.

As seen in Figure 4.6 the equilibrium temperature remains constant with the increase in the injection time. Additionally, the change in the ice thickness, meaning the rate of melt, at the balance temperature phase remains the same. Nevertheless, the span of the heat balance stage increases linearly, resulting in a rise in the overall melting thickness. This suggests that increasing the injection time cannot alter the temperature of the balance. The total volume of melt ice is improved by increasing the length of the balanced temperature stage.

4.2. Summary

Abovementioned simulation model shows that the initial temperature, the flow rate, the initial temperature and the time of injection of the deicing fluid have a momentous weight on the deicing process. The flow rate and the initial temperature of deicing fluids virtually have the same influence over deicing process, both of which can alter the melting rate and total volume of melt ice by modifying the balance temperature.

The injection time can vary the balance temperature phase duration. Nevertheless, the value of the equilibrium temperature cannot be changed while it can only affect the total melt ice volume and has no effect on the rate.

5. CONCLUSION

5.1. A Practical Application of This Study

Purpose of this study is to form a mathematical model of deicing process on a parked aircraft using jet flow.

Adopted approach for this purpose is to analyze the heat transfer characteristic between the elements of the focused systems. Firstly, whole aspect of heat transfer is stated verbally. Secondly, heat transfer processes are formed in a mathematical manner to fully capture the conditions. Then, among these conditions; surface temperature and ice thickness values are chosen as the dependent values. Finally, placing the heat transfer equations into their places on the ice layer-fluid system; a mathematical model is composed.

Next step was to write a code that based on this model. A numerical method called RKF45 is employed to use the mathematical model on FORTRAN.

Finally, mathematical model was put to use to simulate the deicing process in different conditions. This way, every variable's effect on the process could be seen and discussed individually.

On this paper; ground deicing process on aircraft is examined and a mathematical model is formed in order to provide a basis to predict the amount of heat, time and material to achieve a satisfactory melting effect. Using this model, efficiency of the deicing model can be improved greatly.

5.2. My Contribution

In reviewing the literature on deicing process, it is noted while many researches investigate the heat transfer of deicing process; they do not cover the scopes of aircraft deicing; which are variable fluid temperature, fluid flow rate, injection time or fluid chemistry.

Considering these as the independent variables, a mathematical model is created; then a computer code is written based on it, which can simulate the deicing process on many possible conditions.

5.3 Future of This Study

This model can be improved by enabling; other conditions such as wind or humidity, other elements like wing structure or other heat transfer process namely evaporation. In addition, this model focuses on ice accumulation that formed on the ground. However, in the literature review part, different icing types are mentioned. As a future project, this model can be used on different types of icing accumulations.

Additionally, this paper neglects cost analysis. From an airline company's perspective; a deicing model which also takes into account the spent time and material can be created in the future.

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